Inverse problems in seismology: an introduction

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Forward problem

Physical system
Parameterization: discovery of a minimal set of model parameters whose values completely characterize the system (from a given point of view).
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Inverse problem

Model

Physical theory

Predicted measurements
Inverse problem

Model → Physical theory → Predicted measurements → Actual data
Inverse problem: use of the actual results of some measurements of the observable parameters to infer the actual values of the model parameters.
**Inverse problem**

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Inverse pb.: use of the actual results of some measurements of the observable parameters to infer the actual values of the model parameters.
The fact that in realistic experiments a finite amount of data is available to reconstruct a model with infinitely many degrees of freedom necessarily means that the inverse problem is not unique in the sense that there are many models that explain the data equally well. The model obtained from the inversion of the data is therefore not necessarily equal to the true model that one seeks. This implies that for realistic problems, inversion really consists of two steps.

Let the true model be denoted by \( m \) and the data by \( d \). From the data \( d \) one reconstructs an estimated model \( \hat{m} \) this is called the estimation problem.
Apart from estimating a model \( m \) that is consistent with the data, one also needs to investigate what relation the estimated model \( m \) bears to the true model \( m \). In the appraisal problem one determines what properties of the true model are recovered by the estimated model and what errors are attached to it. The essence of this discussion is that:

\[
\text{inversion} = \text{estimation} + \text{appraisal}
\]

It does not make much sense to make a physical interpretation of a model without acknowledging the fact of errors and limited resolution in the model.
In general there are two reasons why the estimated model differs from the true model. The first reason is the non-uniqueness of the inverse problem that causes several (usually infinitely many) models to fit the data.

Technically, this model null-space exits due to inadequate sampling of the model space. The second reason is that real data (and physical theories more often than we would like) are always contaminated with errors and the estimated model is therefore affected by these errors as well. Therefore model appraisal has two aspects, non-uniqueness and error propagation.
Appraisal problem

True model $m$  

Forward 

Data $d$
Appraisal problem

True model $m$ → Forward → Data $d$ → Inversion
Appraisal problem

True model \( m \) → Forward → Data \( d \) → Inversion → Estimated Model \( m \)
Appraisal problem

True model $m$

Forward

Data $d$

Inversion

Estimated Model $m$
Appraisal problem

- True model \( m \)
- Forward
- Data \( d \)
- Inversion
- Estimated Model \( m \)

Appraisal pb.: which properties of the true model are recovered from the estimated model.
Appraisal problem

True model \( m \) → Forward → Data \( d \)

Non-uniqueness + Error propagation

Appraisal pb.: which properties of the true model are recovered from the estimated model.
Science is driven by the feedback between predictions and observations. Most of our knowledge of the Earth’s interior comes from analyzing data collected at the surface. Therefore observations are almost always of an indirect nature, and there exist an ‘inverse’ problem to extract information about the deep interior, e.g. by building an image and ‘seeing’ into the Earth.
Science is driven by the feedback between predictions and observations. Most of our knowledge of the Earth’s interior comes from analyzing data collected at the surface. Therefore observations are almost always of an indirect nature, and there exist an ‘inverse’ problem to extract information about the deep interior., e.g. by building an image and ‘seeing’ into the Earth.

Our understanding of all major features within the Earth, such as the crust, mantle, liquid outer core and solid inner core, as well as the most dynamic parts of the interior such as subduction zones and mantle plumes, came about from the study of indirect measurements made at the surface. Indirect data, such as seismic, magnetic and gravity surveys are also a key tool in the search for hydrocarbon deposits as well as in understanding the contemporary plate tectonic environment.
A key question:

- How do we extract reliable information from multi-faceted and complex geophysical data sets?
- What confidence can be placed in conclusions drawn from those data sets?

The difficulty lies as much in finding the right question to ask, as in finding answers.

**Inverse theory** is the name given to the study of extracting information from indirect measurements. It provides a set of incomplete mathematical, statistical and computational techniques for solving such problems.
What makes Geophysics different from other geosciences?

- Physical properties in the Earth’s interior are retrieved from indirect measurements (observations)
- Physical properties are continuously distributed in a 3D volume, observations are sparsely distributed on the surface of the Earth.
In the geosciences linear inverse problems were the first to be studied in detail. A linear inverse problem arises when the mathematical relationship between observables and unknowns are linear, or assumed to be linear. Pioneering work on linear inverse problems was carried out by Backus and Gilbert (1967, 1968, 1970). They considered linear inverse problems in their most general form, with the unknowns represented by continuous functions of space, rather than a discrete set of parameters. They broke inverse problems up into two parts, known as the existence problem, ‘Does any model exist which fits the available data?’, and the uniqueness problem ‘If so, how unique is that model?’.
Backus and Gilbert showed that there exists a fundamental trade-off between the model variance (the error in unknown model value at any point in a medium) and the model resolution (the degree to which the spatial averaging or blurring occurs). In addition many inverse problems were recognized as non-unique, meaning that an infinite class of solutions exist, each fitting the data equally well. Without extra data or introducing new assumptions there is no reason why one single model should be preferred over any other.
How to deal with non-uniqueness? It can be diminished incorporating into the model:

- Physical constraints
  - Velocities are positives
  - Causality
- Non-informative priors
  - Bayesian inference
- Results from Lab experiments
- Common sense = Talk to the Geologists
Examples: Earthquake location

Data vector $d$:
Traveltimes observed at various (at least 3) stations above the earthquake

Model $m$:
3 coordinates of the earthquake location $(x,y,z)$.

Usually much more data than unknowns: **overdetermined system**
Examples: Reflection Seismology

Data vector $d$:
ns seismograms with nt samples
-> vector length $ns \times nt$

Model $m$:
the seismic velocities of the subsurface, impedances, Poisson's ratio, density, reflection coefficients, etc.
Examples: Seismic Tomography

Data vector $d$:

*Traveltimes* of phases observed at stations of the world wide seismograph network

Model $m$:

3-D seismic velocity model in the Earth’s mantle. Discretization using splines, spherical harmonics, Chebyshev polynomials or simply blocks.

Sometimes 10000s of travel times and a large number of model blocks: underdetermined system
A variety of methods may be used to determine seismic velocity through waveform inversion. Traditionally, the objective function is defined as the misfit between the synthetic seismogram and the data, so the best solution would generate the fit with the absolute minimum error.

Inversions that make use of the full waveform of the seismic signal, including both body waves and surface wave modes, should in principle be superior to methods based on more narrowly selected discrete data, such as arrival times or phase velocities. Waveform inversion also allows us to make use of the information contained in the higher mode signal without having to stack seismograms and identifying modes.
By constructing synthetic seismograms and comparing them to the recorded data we use more of the information in the seismogram, not just the arrival time and first motion data.

Stein and Wyssession, "An Introduction to seismology, earthquakes and Earth structure"
Waveform modeling

Figure 6.3-5: Seismogram as the convolution of the source, structure, and instrument signals.

\[ u(t) = x(t) \ast g(t) \ast q(t) \ast i(t) \]

\[ U(\omega) = M(\omega) \cdot G(\omega) \cdot Q(\omega) \cdot I(\omega) \]

- Source spectra
- Reflections & conversions at interfaces (Green Function)
- Attenuation
- Instrument response

Seismology I - Inverse problem
At one point on the fault slip takes a finite time (called "rise time"): 
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The slip travels along the fault at rupture velocity $v_r$, so there is also a finite “rupture time”
The source time function is the combination of the rise time and the rupture time:

\[ \text{Slip rate} \times \text{Slip rate} = \text{Slip rate} \]

Directionality affects the rupture time
\( e(t) \) represents reflections due to the Earth structure

If modeling only the P arrival, it’s only needed for shallow events
The loss of energy with time

\[ A(t) = A_0 e^{-\omega t/2Q} \]
The response of the seismometer is different for different frequencies so it also filters the data.
Seismic source inversions

The different approaches for seismic source inversion can be grouped according to:

- the source model accepted;
- the type of information used to retrieve the source parameters within the model selected;
- the frequency content of the input data which reflects combined effect of the magnitude and distance of the seismic event.

The frequency content is a principal feature of seismic data according to which inverse methods are designed.

Two main approaches: (1) point source approximation (the source extension is small compared to the wavelength); (2) finite extension source.
At present, the moment tensor description of a seismic source is the most widely used in the framework of seismic source inversion at point source approximation. The moment tensor description is more general with respect to a simple double-couple description.

In addition to being an elegant representation of the source, the moment tensor has two advantages for source studies. First, it allows us to analyze seismograms without assuming that they result from slip on a fault. In some applications, such as deep earthquakes or volcanic earthquakes, we would like to identify possible isotropic or CLVD components. Second, the moment tensor makes it easier to invert seismograms to find source parameters.
The linearity between the moment tensor and Green’s functions can be used for calculating moment tensor elements from observations (moment tensor inversion).

\[ u_k(t) = \sum_{i,j=1}^{3} M_{ij}(t) \ast G_{ki,j}(t) \]

There are various methods of inversion for moment tensor elements. The inversion can be done in the time or frequency domain. Different data (e.g. free oscillations, surface and body waves; different seismogram components) can be used.
In the time domain, the vector $d$ consists of $n$ sampled values of the observed ground displacement at various arrival times, stations, and azimuths. $G$ is a $n \times 6$ matrix containing the Green’s functions calculated using an appropriate algorithm and earth model, and $m$ is a vector containing the 6 moment tensor elements to be determined. In the frequency domain the equation can be written separately for each frequency.

\[ d = Gm \]
For what focal mechanisms and moment tensors may be used?

- find the rupture plane of the earthquake
- input for structural studies
- indicate different style of failure (non-double-couple components)
- determine stress principal axes or average regional strain tensor
Seismic waves provide information about the Earth’s deep interior, and their predictable behaviour makes it possible to obtain high-resolution models of some of the Earth’s internal properties.

Most of the major features of the Earth were discovered in the early days of instrumental seismology from the last century. The Earth’s fluid core was discovered in 1906 by Oldham from analysis of seismograms from the 1897 Assam, India, earthquake. Mohorovicic determined the boundary between the crust and mantle in 1911 from analysis of seismograms of the 1909 Kupatal earthquake in the Balkans. Lehmann discovered the Earth’s inner core in 1936.
Byerly discovered the 20° discontinuity in a study of travel times from the 1925 Helena, Montana, earthquake, but it was not until 1966 that Johnson and Anderson, using seismic array data, showed that the 20° discontinuity was the result of two upper mantle discontinuities, one at about 410 km depth and another at about 600 km depth.

Seismologists have exerted a great effort to develop and refine a one-dimensional model for use in routine earthquake location. Nevertheless, we know that some lateral heterogeneity exists at every depth in the Earth. Methods have thus been developed to use seismological data to investigate laterally heterogeneous structure.

The primary effort at present is to determine the 3-D structure of the Earth, commonly called seismic tomography. The 3-D structure is determined in terms of perturbations from a standard reference Earth model, usually a spherically symmetric Earth model.
**Tomography** = the process of forming images of the interior of an object from measurements made along rays passed through that object ("tomo" comes from the Greek word for "slice").

The **Computerized Axial Tomography (CAT)** scanner is an x-ray imaging device. The scanner solves an inverse problem for the x-ray opacity of body tissues using measurements of the amount of radiation absorbed from many criss-crossing beams of x rays. The basic physical model underlying this device is the idea that the intensity of x rays diminishes with the distance travelled, at a rate proportional to the intensity of the beam, and an absorption coefficient that depends on the type of tissue:

\[
dI/ds = -\mu(x, y)I
\]

I is the intensity of the beam, s the distance along the beam, and \(\mu(x, y)\) the absorption coefficient, which varies with position.
Medical applications

X-ray absorption & scattering

Tissues and bones have \( \neq \) absorption and scattering coefficients \( \mu(x, y) \).

Recorded intensity goes as

\[
I = I_0 \exp \left( \int_{\text{ray}} -\mu(x, y) \, ds \right).
\]

Sources and detectors rotate to achieve perfect “coverage”.

"Seismology I - Inverse problem"
An idealized x-ray tomography device measures x-ray absorption along lines passing through the body. After a set of measurements is made, the source and detectors are rotated and the measurements are repeated, so data along many criss-crossing lines are collected. The inverse problem is to determine the x-ray opacity as a function of position in the body.

Similarly ...

**Seismic tomography** = the process of forming images of the interior of the Earth from measurements made along rays (seismic waves) passed through the Earth.
Seismic tomography depends on the contrast in seismic properties. Such differences in 3-D structure are reflected directly in the times of arrival of different seismic phases or in the shape and amplitude of the seismic waveform. The normal procedure is to examine the departure of the observed properties of the seismic wavefield from the predictions from a reference model. In most cases the significant quantity is a time shift in the arrival of a phase. The travel time residuals for body waves or modified dispersion characteristics for surface waves can then be used as the basis for recovering the 3-D model.
The influence of velocity structure on seismic waves: a slow wave speed region (shaded) surrounded by a normal wave speed material with two propagation paths from seismic sources (A and B) to a common seismograph (R).

For body waves the propagation difference appears as a shift in the arrival time of a common seismic phase; for surface waves as a frequency-dependent phase shift of the waveforms.
Considering this simple case of a slow wave speed region surrounded by normal wave speed material, the waves propagating to the receiver R will have different characteristics depending on whether they traverse the slow or normal regions. For body waves, the travel time for path A, $t_A$, will be longer than travel time for path B, $t_B$.

We compare these arrival times to the travel time along same path but through a reference model designated $t_{rA}$ and $t_{rB}$. The differential times:

$$\Delta t_A = t_A - t_{rA}$$
$$\Delta t_B = t_B - t_{rB}$$

provide a comparative measure of the influence of the structure along the two different propagation paths.
A number of general concepts can be illustrated with a simple discrete linear inverse problem, such as seismic travel time tomography. We have a single seismic ray passing through a two parameter block model. The unknowns are the changes in slownesses (reciprocal of seismic velocity) in the blocks from a homogeneous slowness model \((S_1;S_2)\). The single datum, \(t_1\), is the difference between the observed travel time of the ray and that calculated in the reference (homogeneous) slowness model. The question is can we find the slownesses in the blocks?
Linearization of the seismic travel time equation gives

\[ \Delta t_1 = L_1 \Delta S_1 + L_2 \Delta S_2 \]

where \( L_j \) is the length of the ray in the \( j \)-th block. Clearly we have one linear equation and two unknowns, and hence no unique solution. In fact there is a complete trade-off between the slowness variables, and only the average slowness of the two blocks is constrained by the data.
Superficially the situation B looks much better, but this is deceptive. Regardless of the fact that we now have many rays traversing the blocks, it’s straightforward to show that each ray contributes an equation which is just a scalar multiple of A. Hence we really still have only one independent equation, even though we now have many rays. If the data are error free the situation B is identical to A and no extra information is present. (Note: If the data contained noise then there would be a benefit from the averaging effect of the rays, but the again only the average slowness would be constrained.)
The situation C is entirely different. Here we have two equations that pass through different sides of the blocks. Hence the ratio of the lengths is no longer equal

\[ \frac{l_1}{l_2} \neq \frac{l_3}{l_4} \]

This means that the two constraint equations, are linearly independent, and hence can be solved uniquely for the slownesses (S₁;S₂).
The situation D, is an improvement of C, in that now many rays from different directions are present. Here we again can solve for the two unknowns and would be likely to do so with less variance in the model parameters when the data contained noise (again due to the combined effect of the many rays).

In the parlance of linear algebra, situations A and B lead to an under-determined linear system of equations; C to an even determined; and D to an overdetermined.
In situation E we have the same rays as in D but have chosen to use more blocks to represent the slowness field. Close inspection shows that all the previous cases hold simultaneously in E and hence this is what is known as a mixed determined problem, containing both over and under determined parameters. Comparing D to E we see that as we seek to constrain slowness variations over shorter spatial scales our model variance increases. This is because with noisy data even the best constrained slowness blocks in E will have larger error than the two unknowns in E. Hence there is a trade-off between model variance (error) and spread of resolution (inverse of cell size). E illustrates this trade-off, which is a general property of all linear inverse problems.
This simple linear example illustrates many important concepts of inverse problems, namely that the mere number of data is unimportant, but the (linear) independence of the data matters most; that non-uniqueness is often present and parameterization is a choice; that problems are often mix-determined; and that a trade-off between resolution and variance can not be avoided. For discrete inverse problems with numbers of unknowns less than $10^3$, it is practical to calculate quantities like the model covariance matrix and the model resolution matrix, which characterize these properties.
The observed travel times for body waves are influenced by the structures around the ray path and the level of heterogeneity in most parts of the Earth is such that the deviation in ray paths due to the structure is significant.

Since each ray carries with it a measure of the integrated perturbations in seismic slowness along the propagation path, if there is a dense enough sampling of the region by crossing ray paths it is possible to determine the heterogeneity distribution.

Blocks of uniform velocity perturbation traversed by a ray.
1. There are several ways to parameterize the 3-D velocity perturbations of the Earth model. The model can be divided into blocks of uniform velocity perturbation, or spherical harmonic functions can be used in global models.

2. Each travel time residual is then associated with a raypath connecting the source and the receiver.

3. Find the travel time through each block that the ray crosses. The total travel time perturbation along the ray path is the sum of the block travel time and the relative velocity perturbation within the block.

\[ r = \sum_{k} b_k v_k \]

- \( b_k \) = the ray travel time through the \( k \)th block
- \( v_k \) = the velocity perturbation for the \( k \)th block
If we set each block not encountered by the ray to zero this can be written as:

\[ r = \sum_{k=1}^{m} b_k v_k \Rightarrow \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_n \end{pmatrix} = \begin{pmatrix} 0 & 0.2 & 0 & 0 & \cdots \\ 1.3 & 0 & 0.3 & 0 & \cdots \\ 0 & 0 & 0 & 0.1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0.1 & 0 & 0 & \cdots \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_m \end{pmatrix} \Rightarrow d = Gm \]
Seismic tomography is analogous to medical imaging. However, the seismic case is more complicated than the medical case for several reasons.

1. Seismic ray paths are not straight and are a function of the velocity model
2. The distribution of the seismic sources and receivers is sparse and non-uniform
3. The earthquake locations are not well known
4. Timing errors are common in seismic data
The resolution of a tomographic image is the highest spatial frequency at which the image is expected to be meaningful. In other words, it is the smallest spatial extent of a velocity anomaly, that the inverted seismic database can properly map. The resolution is limited by two factors: the quality of the data and the uniformity of their spatial distribution (the “data coverage”), and the approximations involved in the formulation of the inverse problem.

The resolution of a tomographic model is often evaluated using an impulse response or checkerboard test: a synthetic set of travel times are created for a simple velocity model using the same set of raypaths present in the real data; the synthetic travel times are then inverted to see how well the starting model is recovered.
COMPLICATIONS WITH TOMOGRAPHIC MODELS

1. Trade-off between velocity anomalies and earthquake locations
2. Unresolved shallow structure
3. Improper sampling and smearing
4. Deviation of the raypath from that in the reference model
5. Ray theory assumptions

A lack of rays crossing each other from different directions can lead to smearing of the velocity anomaly.
Seismology I - Inverse problem

(using body waves travel times and normal mode frequencies)

http://mahi.ucsd.edu/Gabi/rem.html
Adams-Williamson Equation

- We can use $V_p$ and $V_s$ to get density, defining the seismic parameter
  \[ \Phi = V_p^2 - \frac{4}{3} V_s^2 = \frac{K}{\rho} \]
  \[ K = \rho \frac{dP}{d\rho} \]

- This gives us the Adams-Williamson equation:
  \[ \frac{d\rho(r)}{dr} = - \frac{\rho(r)g(r)}{\Phi(r)} \]

- The equation is useful because we know $\Phi(r)$ (from the seismic velocity observations)

- So we can assume a surface density and gravity and then iteratively calculate $\rho(r)$ at successively greater depths using our observations of $\Phi(r)$. 
Interpretation of the actual chemistry, physical state, and dynamic behaviour associated with the seismological structure requires experimental and modelling results from other disciplines such as mineral physics and geodynamics. Nonetheless, seismology has the primacy of providing our best resolution of the actual structure of the planet.
The processes operating near the surface of the Earth have produced a remarkably complex crustal structure. Our detailed knowledge of the Earth structure generally diminishes with depth.

Below the Moho, the crust mantle boundary, our detailed resolution of internal structure diminishes, but surprisingly we obtain increasingly complete global coverage. This is because earthquakes provide the primary seismic sources rather than human-made explosion sources. While both earthquake and seismic station distributions are spatially nonuniform, there are vast numbers of paths through the Earth, yielding fairly complete global coverage. The velocity structure of the upper mantle is very complex, with strong lateral variations associated with the deep structure of plate tectonics processes and depth variations associated with a myriad of high-pressure phase transformations in mantle minerals. The deeper layers of the Earth, the lower mantle and core, are well characterized in their average properties as a function of depth.
Structure of the Earth
1. Inner core: depth of 5150-6370 km
2. Outer core: depth of 2,890-5,150 km
3. D" layer: depth of 2,700-2,890 km
4. Lower mantle: depth of 650-2,890 km
5. Transition region: depth of 400-650 km
6. Upper mantle: depth of 10-400 km
7. Oceanic crust: depth of 0-10 km
8. Continental crust: depth of 0-75 km
Schematic view of the interior of Earth. 1. continental crust - 2. oceanic crust - 3. upper mantle - 4. lower mantle - 5. outer core - 6. inner core

A: Mohorovičić discontinuity - B: Gutenberg Discontinuity - C: Lehmann discontinuity
The first efforts: 1910, Mohorovičić first identified and abrupt increase in velocity beneath the shallow rocks under Europe. The boundary separating mantle rocks is now called the Moho and is a ubiquitous boundary of highly variable character. Although we generally accept that the crust is chemically distinct from the upper mantle and that the Moho likely involves a chemical contrast, additional contributions to the seismically detectable boundary may arise from transitions in rheological properties, phase transitions in shallow mineral structures, and petrographic fabrics of the rocks. These complexities are combined with the complex tectonic history of the surface to provide a remarkably heterogeneous crustal layer.

A feature common to all crustal environments: shallow rocks have slower seismic velocities than deeper rocks, usually approximating a low-velocity layer over a faster mantle.
Continental crust: the thickness varies from 20 to 70 km.

Oceanic crust: from 5 to 15 km.

Both regions have very low seismic velocity surface cover, with the water and mud layers on oceanic crust having a particularly low velocity. Both regions tend to have at least two crustal subdivisions. The variation in crustal thickness between oceanic and continental regions ensures a very different appearance of the seismograms as a function of distance for the two regions. The crust tends to be thicker under tectonically stable regions and old mountain belts and thinner under actively rifting areas.
Oceanic crust: depth of 5-15 kilometres - The majority of the Earth's crust was made through volcanic activity. The oceanic ridge system, a 40,000 kilometre network of volcanoes, generates new oceanic crust at the rate of $17 \text{ km}^3/\text{per year}$, covering the ocean floor with an igneous rock called basalt. Hawaii and Iceland are two examples of the accumulation of basalt islands.

Continental crust: depth of 20-70 kilometres - This is the outer part of the Earth composed essentially of crystalline rocks. These are low-density buoyant minerals dominated mostly by quartz ($\text{SiO}_2$) and feldspars (metal-poor silicates). The crust is the surface of the Earth. Because cold rocks deform slowly, we refer to this rigid outer shell as the lithosphere (the rocky or strong layer).
As we delve deeper into the Earth, the information gleaned from seismology plays an increasingly large role in our knowledge of the interior. Only a handful of unusual processes have exposed samples of mantle materials at the surface for petrological analysis, and no drill hole has yet penetrated the Moho anywhere. Thus the material properties revealed by seismology play a dominant role in constraining both the composition and dynamics of the mantle.
Additional stratification of the mantle is represented by global seismic-velocity boundaries at depths near 410, 520, and 660 km that define the transition zone in the lower part of the upper mantle. These boundaries give rise to reflections and conversions of seismic waves, which reveal the boundaries and allow us to model them in terms of depth and contrasts in velocity, density, and impedance. The 410-, 520-, and 660-km discontinuities probably represent mineralogical phase transformations. The upper 250 km of the mantle is particularly heterogeneous, with strong regional variations associated with surface tectonic provinces.
The uppermost mantle just below the Moho is a region with high seismic velocities (P 8.0-8.5 km/s) that is often called the lid because it overlies a lower-velocity region. The base of the lid may be anywhere from 60 to 200 km in depth, and it is thought to represent the rheological transition from the high-viscosity lithosphere to the low-viscosity asthenosphere. The thickness of the lid varies with tectonic environment, generally increasing since the time of the last thermotectonic event. Under very young ocean crust, the lid may actually be absent, but it is commonly observed under old ocean plates. Beneath the lid is a region of reduced velocity, usually referred to as the low-velocity zone (LVZ).
The LVZ may be very shallow under ridges, and it deepens as the lid thickness increases. Beneath old continental regions, the LVZ may begin at a depth of 200 km, with relatively low velocities usually ending by 330 km, where a seismic discontinuity is intermittently observed. A relatively strong seismic discontinuity is observed under some continental and island-arc regions near a depth of 220 km, which may be associated with LVZ structure or some transition in mantle fabric associated with concentrated LVZ flow structures.

A depth of 710 km or so is reasonable for the boundary between the upper and lower mantle, as known mineralogical phase transformations could persist to this depth.
Transition region: depth of 400-650 kilometres

The transition region or mesosphere (for middle mantle), sometimes called the fertile layer and is the source of basaltic magmas. It also contains calcium, aluminium, and garnet, which is a complex aluminium-bearing silicate mineral. This layer is dense when cold because of the garnet. It is buoyant when hot because these minerals melt easily to form basalt which can then rise through the upper layers as magma.

Upper mantle: depth of 10-400 kilometres

Solid fragments of the upper mantle have been found in eroded mountain belts and volcanic eruptions. Olivine (Mg,Fe)\(_2\)SiO\(_4\) and pyroxene (Mg,Fe)SiO\(_3\) have been found. These and other minerals are crystalline at high temperatures. Part of the upper mantle called the asthenosphere might be partially molten.
The standard seismological procedure for studying the lower mantle has involved inversion of travel-time curves for smoothly varying structure. There is absence of significant boundaries throughout much of the vast region from 710 to 2600 km. In the lowermost 250 km of the mantle there is a region of anomalous velocity gradients called the D'' region. Smooth variations in properties in the lower mantle.
D" layer: depth of 2,700-2,890 kilometres
This layer is 200 to 300 kilometres thick. Although it is often identified as part of the lower mantle, seismic evidence suggests the D" layer might differ chemically from the lower mantle lying above it. Scientists think that the material either dissolved in the core, or was able to sink through the mantle but not into the core because of its density.

Lower mantle: depth of 650-2,890 kilometres
The lower mantle is probably composed mainly of silicon, magnesium, and oxygen. It probably also contains some iron, calcium, and aluminium. Scientists make these deductions by assuming the Earth has a similar abundance and proportion of cosmic elements as found in the Sun and primitive meteorites.
The Earth's core was first discovered in 1906 when Oldham found a rapid decay of P waves beyond distances of 100°, and he postulated that a low-velocity region in the interior produces a shadow zone. Gutenberg accurately estimated a depth to the core of 2900 km in 1912, and by 1926 Jeffreys showed that the absence of S waves traversing the core required it to be fluid. The core extends over half the radius of the planet and contains 30% of its mass. The boundary between the mantle and core is very sharp and is the largest compositional contrast in the interior, separating the molten core alloy from the silicate crystalline mantle. The material properties of the core are quite uniform, with a smoothly increasing velocity structure down to a depth of 5150 km, where a sharp boundary separates the outer core from a solid inner core. Decrease in P velocity from values near 13.7 km/s at the base of the mantle to around 8 km/s at the top of the outer core.
Inner core: depth of 5,150-6,370 kilometres
The inner core is made of solid iron and nickel and is unattached to the mantle, suspended in the molten outer core. It is believed to have solidified as a result of pressure-freezing which occurs to most liquids under extreme pressure.

Outer core: depth of 2,890-5,150 kilometres
The outer core is a hot, electrically conducting liquid (mainly Iron and Nickel). This conductive layer combines with Earth's rotation to create a dynamo effect that maintains a system of electrical currents creating the Earth's magnetic field. It is also responsible for the subtle jerking of Earth's rotation. This layer is not as dense as pure molten iron, which indicates the presence of lighter elements. Scientists suspect that about 10% of the layer is composed of sulphur and oxygen because these elements are abundant in the cosmos and dissolve readily in molten iron.
Structure of the Earth

Lithosphere: Crust and top of mantle, which deform elastically under vertical crustal loading. Oceanic Lithosphere thickens to c. 100km with age. Continental lithosphere is only a little thicker.

Asthenosphere: Region beneath Lithosphere, which "flows" to permit isostatic compensation. It probably extends deep into the mantle, but the term is usually reserved for the least viscous part, just below the lithosphere, c. 100km thick.

Crust, Mantle and Core

<table>
<thead>
<tr>
<th>Depth, Km</th>
<th>Density, kg m⁻³ (x 10³)</th>
<th>% of Earth's mass</th>
<th>Seismic Velocities Vp and Vs, km s⁻¹</th>
<th>Physical properties of regions</th>
<th>Compositions of regions</th>
<th>Nature of boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25-90</td>
<td>0.7%</td>
<td>LOW VELOCITY ZONE (few % partial melt, more prominent below oceans)</td>
<td>DEPLETED PERIDOTITE</td>
<td>Compositional</td>
<td>PHASE CHANGE TO HIGH-PRESSURE POLYMORPHS</td>
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<tr>
<td>10</td>
<td>average: 35</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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</table>

Tectosphere: Crust and uppermost mantle, which moves laterally as a plate. Oceanic tectosphere is considerably thicker, 400km, or even more.
Seismic waves provide a probe of the Earth’s deep interior, and their predictable behaviour makes it possible to obtain high-resolution models of some of the Earth’s internal properties.

A model is a simplified mathematical representation of the actual three-dimensional material property variations within the planet.

Seismology provides primary constraints on the variations of density, rigidity, and incompressibility and secondary constraints on the temperature field at all depths in the Earth.
Ray theory - WKBJ approximation - For high frequency body waves, the travel time is the integral of the time increments along the propagation path, in terms of the local wave speed slowness $\alpha^{-1}$:

$$t_A = \int_{rayA} \frac{ds}{v(r)} = \int_{rayA} ds \alpha^{-1}(r)$$

If the deviations of the 3-D structure from the reference model are not large, the difference of the ray paths in the 3-D and reference structure can be ignored. For this small perturbation approximation, the travel time residual $\Delta t_A$ is the integrated difference in slowness between the actual Earth structure and the reference model along the unperturbed ray path.

$$\Delta t_A = \int_{rayA} ds \left( \alpha^{-1}(r) - \alpha_r^{-1}(r) \right)$$
We describe a method to invert surface wave group or phase velocity measurements to estimate 2-D models of the distribution and strength of velocity variations.

Using ray theory, the forward problem for surface wave tomography consists of predicting a frequency dependent travel time $t_{R/L}(\omega)$. For both Rayleigh (R) and Love (L) waves from a set of 2-D phase or group velocity maps, $c(r, \omega)$:

$$t_{R/L}(\omega) = \int_{\text{ray}} c_{R/L}^{-1}(r, \omega) ds$$

Where $r=[\Theta, \phi]$ is the surface position vector, $\Theta$ and $\phi$ are colatitude and longitude, and ray specifies the path.
The dispersion maps are nonlinearly related to the seismic structure of the earth, $M(r,z)$. If we assume that the model is isotropic, $M(r,z) = [v_s(z), v_p(z), \rho(z)]$ is the position dependent structure vector composed of the shear and compressional velocities and densities.

By surface wave tomography we mean the use of a set of observed travel times $t^{obs}(\omega)$ for many different paths to infer a group or phase velocity map, $c(r)$, at frequency $\omega$.

Using a 2-D reference map, $c_0(r)$, the travel time perturbation relative to the prediction from $c_0(r)$ is:

$$
\delta t = t - t_0 = \int_{\text{ray}} \frac{ds}{c} - \int_{\text{ray}} \frac{ds}{c_0} = \int_{\text{ray}} \frac{m}{c_0} ds
$$

$$
m = \frac{c_0 - c}{c}
$$
Our goal is to estimate the vector function \( m(r) = [m_o(r), \ldots, m_n(r)] \) using a set of observed travel time residuals \( d \) relative to the reference model \( c_o(r) \):

\[
d = \delta t_i = t_i - t_{0i} = \int_{ray \, c_0}^m \frac{m}{ds} + \varepsilon
\]

Surface wave tomography (Ditmar and Yanovskaya, 1987; Yanovskaya and Ditmar, 1990). The methodology is an extension of the classical 1D Backus and Gilbert inversion approach.
Global scale

RAYLEIGH WAVE 35s
RAYLEIGH WAVE 50s

RAYLEIGH WAVE 100s

The reliability of the group velocity maps across large regions degrades sharply below 15 s and above 150-200 s for Rayleigh waves and 100-125 s for Love waves. Surface waves maps at and below 30 s period are particularly important because they provide significant constraints on crustal thickness by helping to resolve Moho depth from the average shear velocity of the crust. Although there have been numerous studies of surface wave dispersion that have produced measurements of group and/or phase velocities between 10 and 40 s period, these studies have typically been confined to areas of about 15° or less in lateral extent.

Phase and group velocity maps provide constraints on the shear velocity structure of the crust and uppermost mantle. Accurate high-resolution group velocity maps, in particular, are useful in monitoring clandestine nuclear tests.
Measurements of group velocities are much less sensitive to source effects than phase velocities because they derive from measurements of the wave packet envelopes rather than the constituent phases. This is particularly true at shorter periods and longer ranges. Group velocity sensitivity is compressed nearer to the surface than the related phase velocities, which should provide further help in resolving crustal from mantle structures.