SEISMOLOGY I

Laurea Magistralis in Physics of the Earth and of the Environment

Seismic Surface waves

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Surface Waves and Free Oscillations

Surface waves in an elastic half spaces: Rayleigh waves

- Potentials
- Free surface boundary conditions
- Solutions propagating along the surface, decaying with depth
- Lamb's problem

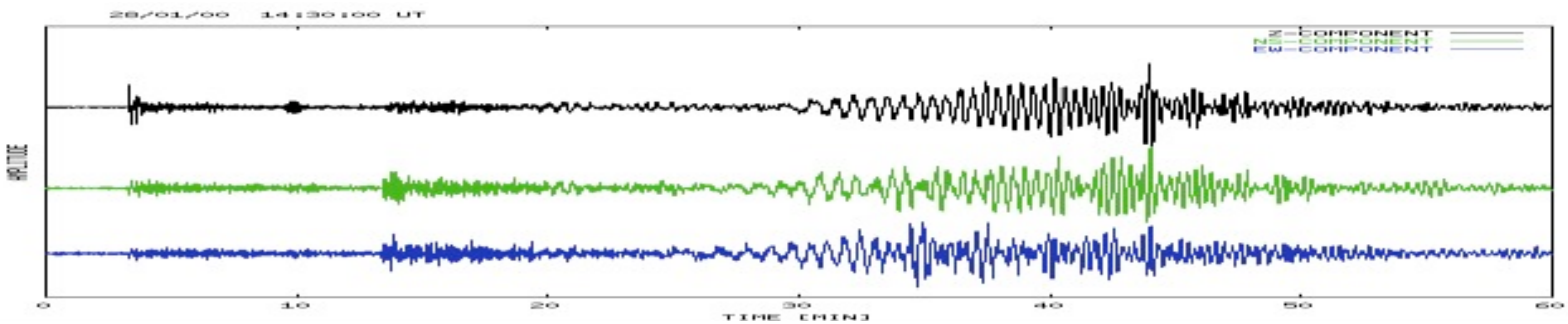
Surface waves in media with depth-dependent properties: Love waves

- Constructive interference in a low-velocity layer
- Dispersion curves
- Phase and Group velocity

Free Oscillations

- Spherical Harmonics
- Modes of the Earth
- Rotational Splitting

Data Example



Question:

We derived that Rayleigh waves are non-dispersive!

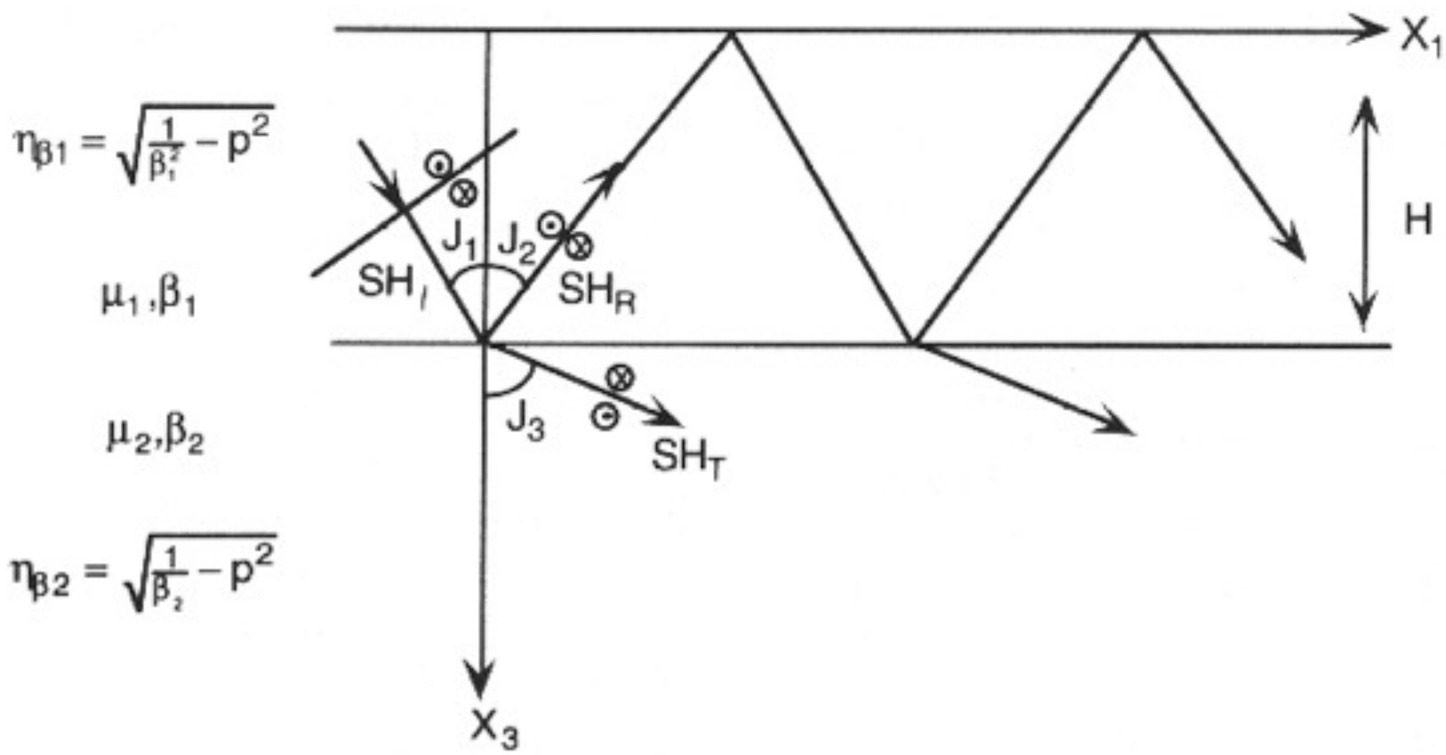
But in the observed seismograms we clearly see a highly dispersed surface wave train?

We also see dispersive wave motion on both horizontal components!

Do SH-type surface waves exist?
Why are the observed waves dispersive?

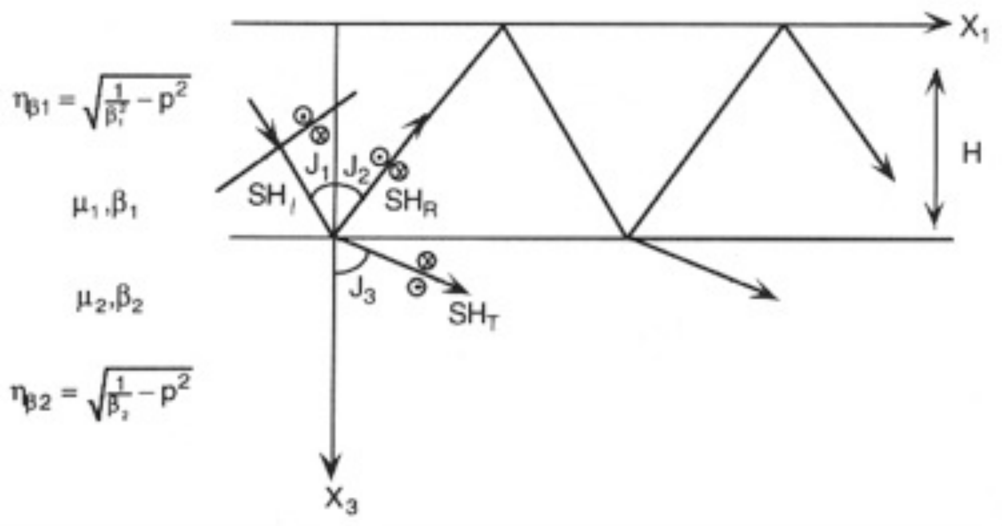
Love Waves: Geometry

In an elastic half-space no SH type surface waves exist. Why?
Because there is total reflection and no interaction between an evanescent P wave and a phase shifted SV wave as in the case of Rayleigh waves. What happens if we have a layer over a half space (Love, 1911) ?



Repeated reflection in a layer over a half space.
Interference between incident, reflected and transmitted SH waves.
When the layer velocity is smaller than the halfspace velocity, then there is a critical angle beyond which SH reverberations will be totally trapped.

Love waves: trapping - 1



$$u_{y1} = A\exp[i(\omega t + \omega\eta_{\beta 1}z - kx)] + B\exp[i(\omega t - \omega\eta_{\beta 1}z - kx)]$$

$$u_{y2} = C\exp[i(\omega t - \omega\eta_{\beta 2}z - kx)]$$

$$k = k_x = \frac{\omega}{c}; \quad \omega\eta_\beta = k_z = \frac{\omega}{c}\sqrt{\frac{c^2}{\beta^2} - 1} = kr_\beta$$

$$u_{y1} = A\exp[i(\omega t + kr_{\beta 1}z - kx)] + B\exp[i(\omega t - kr_{\beta 1}z - kx)]$$

$$u_{y2} = C\exp[i(\omega t - kr_{\beta 2}z - kx)]$$

Love waves: trapping - 2

The formal derivation is very similar to the derivation of the Rayleigh waves. The conditions to be fulfilled are:

1. Free surface condition
2. Continuity of stress on the boundary
3. Continuity of displacement on the boundary
4. No radiation in the halfspace

1. $$\sigma_{zy1}(0) = \mu_1 \left.\frac{\partial u_{y1}}{\partial z}\right|_0 = ikr_{\beta 1}\left\{A\exp[i(\omega t - kx)] - B\exp[i(\omega t - kx)]\right\} = 0$$

2. $$\sigma_{zy1}(H) = \mu_1 \left.\frac{\partial u_{y1}}{\partial z}\right|_H = \sigma_{zy2}(H) = \mu_2 \left.\frac{\partial u_{y2}}{\partial z}\right|_H$$

3. $$u_{y1}(H) = u_{y2}(H)$$

4. $$\lim_{\infty} u_{y2}(z) = 0 \quad \text{i.e.} \quad c < \beta_2 \quad \text{i.e.} \quad r_{\beta 2} = -i\sqrt{1 - \frac{c^2}{\beta_2^2}}$$

Love waves: trapping - 3

We obtain a condition for which solutions exist. This time we obtain a frequency-dependent solution a **dispersion** relation

$$\tan\left(H\omega\sqrt{1/\beta_1^2 - 1/c^2}\right) = \frac{\mu_2\sqrt{1/c^2 - 1/\beta_2^{\,2}}}{\mu_1\sqrt{1/\beta_1^2 - 1/c^2}}$$

... indicating that there are only solutions if ...

$$\beta_1 < c < \beta_2$$

Love Waves: Solutions

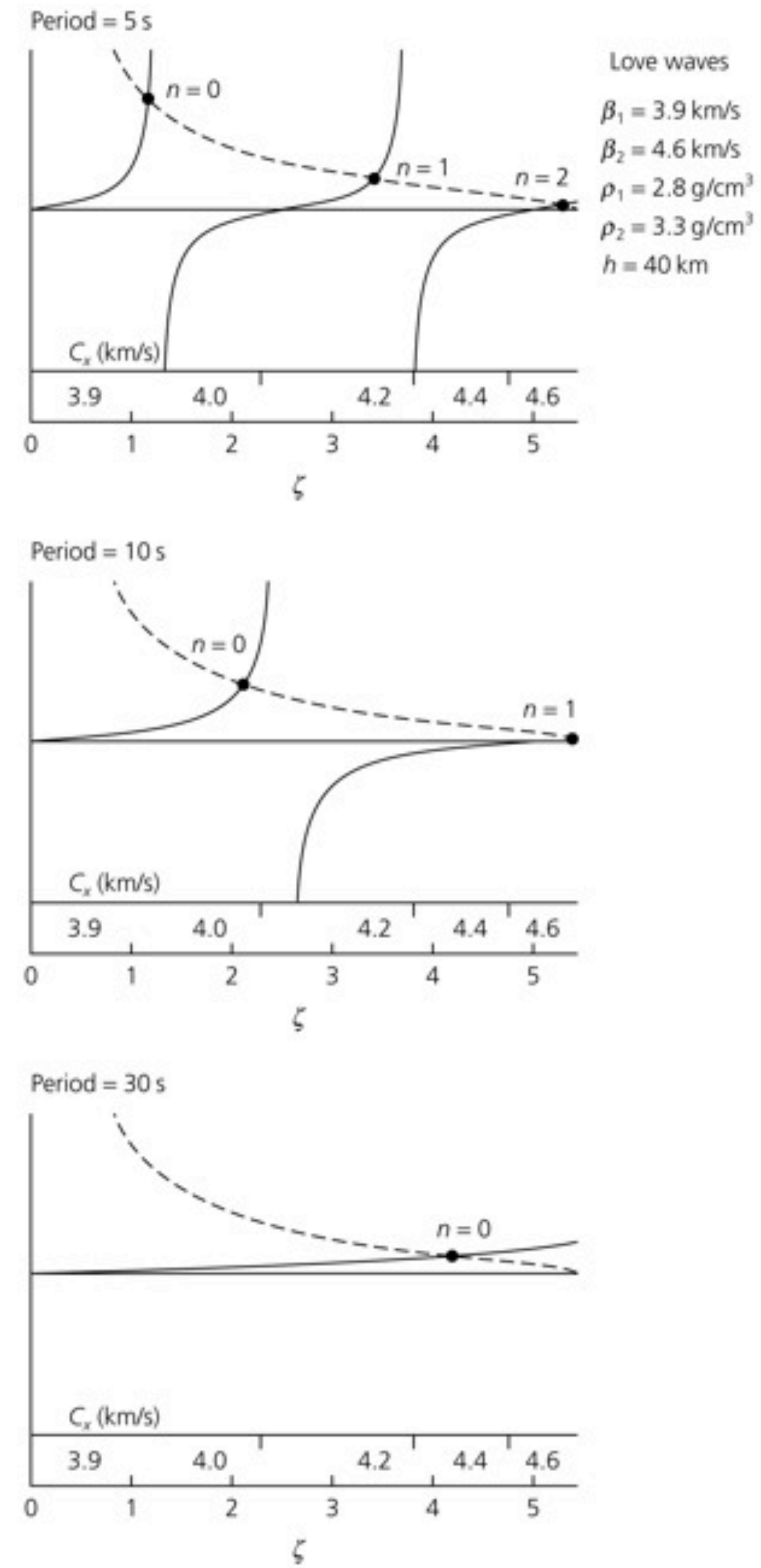
Graphical solution of the previous equation. Intersection of dashed and solid lines yield discrete **modes**.

$$\tan\left(H\omega\sqrt{1/\beta_1^2 - 1/c^2}\right) = \tan(\omega\zeta)$$

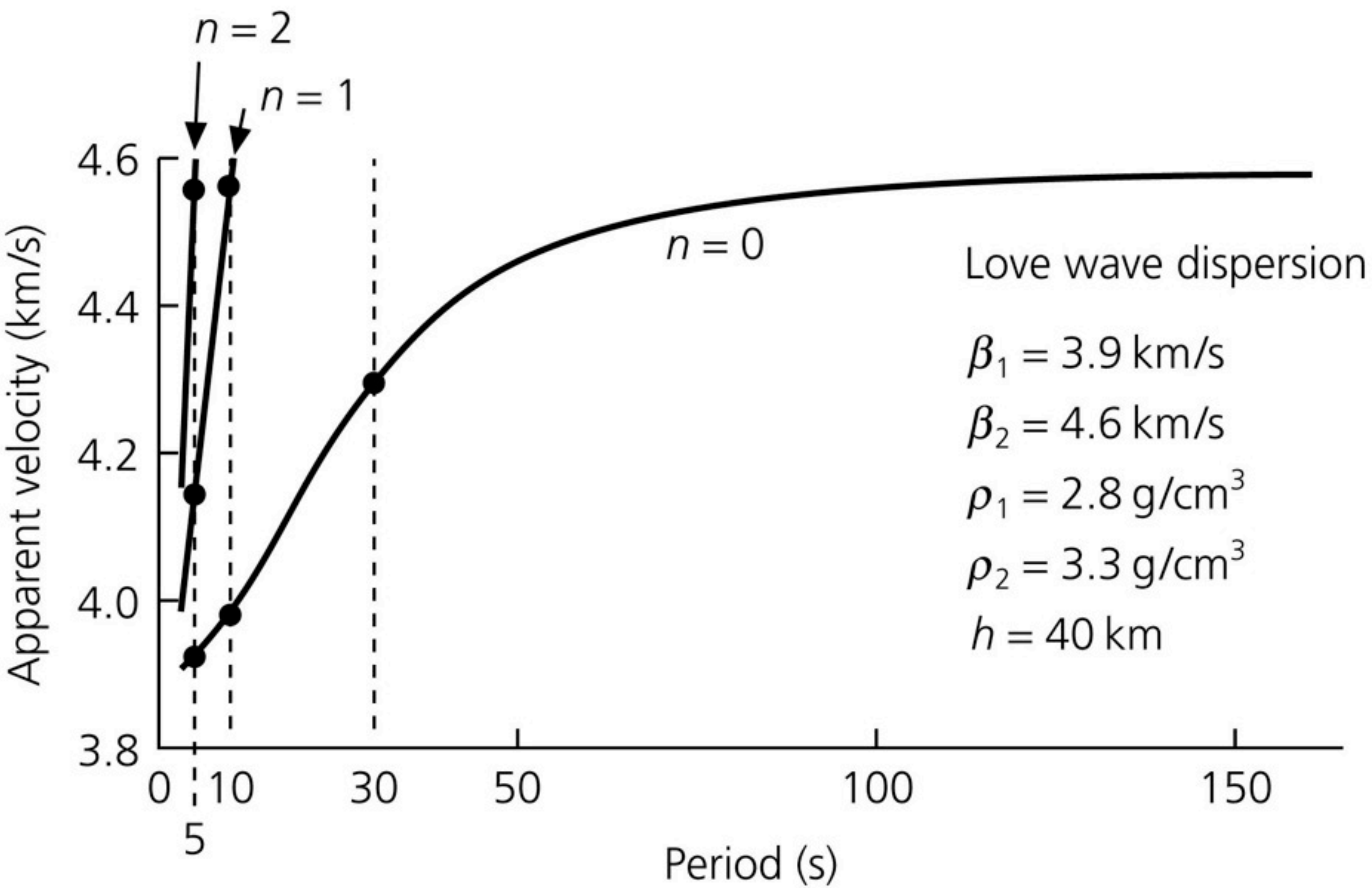
that vanishes when $\zeta = n\dfrac{\pi}{\omega}$

New modes appear at **cut-off frequencies**

$$\omega_n = \frac{n\pi}{H\left(\dfrac{1}{\beta_1^2} - \dfrac{1}{\beta_2^2}\right)^{1/2}}$$



Love Waves: Solutions

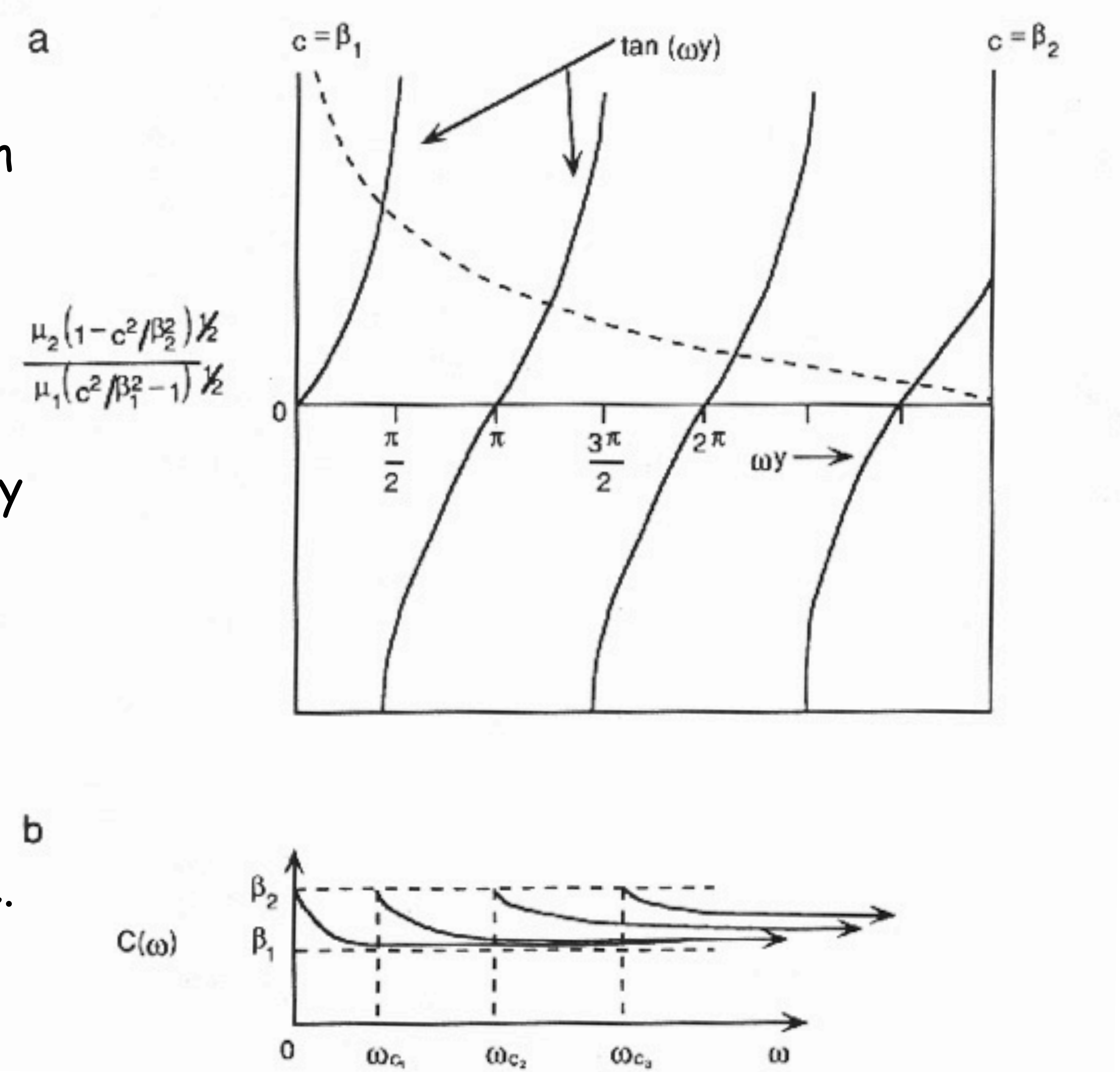


Love Waves: Solutions

Graphical solution of the previous equation. Intersection of dashed and solid lines yield solutions while frequency is varying: discrete modes.

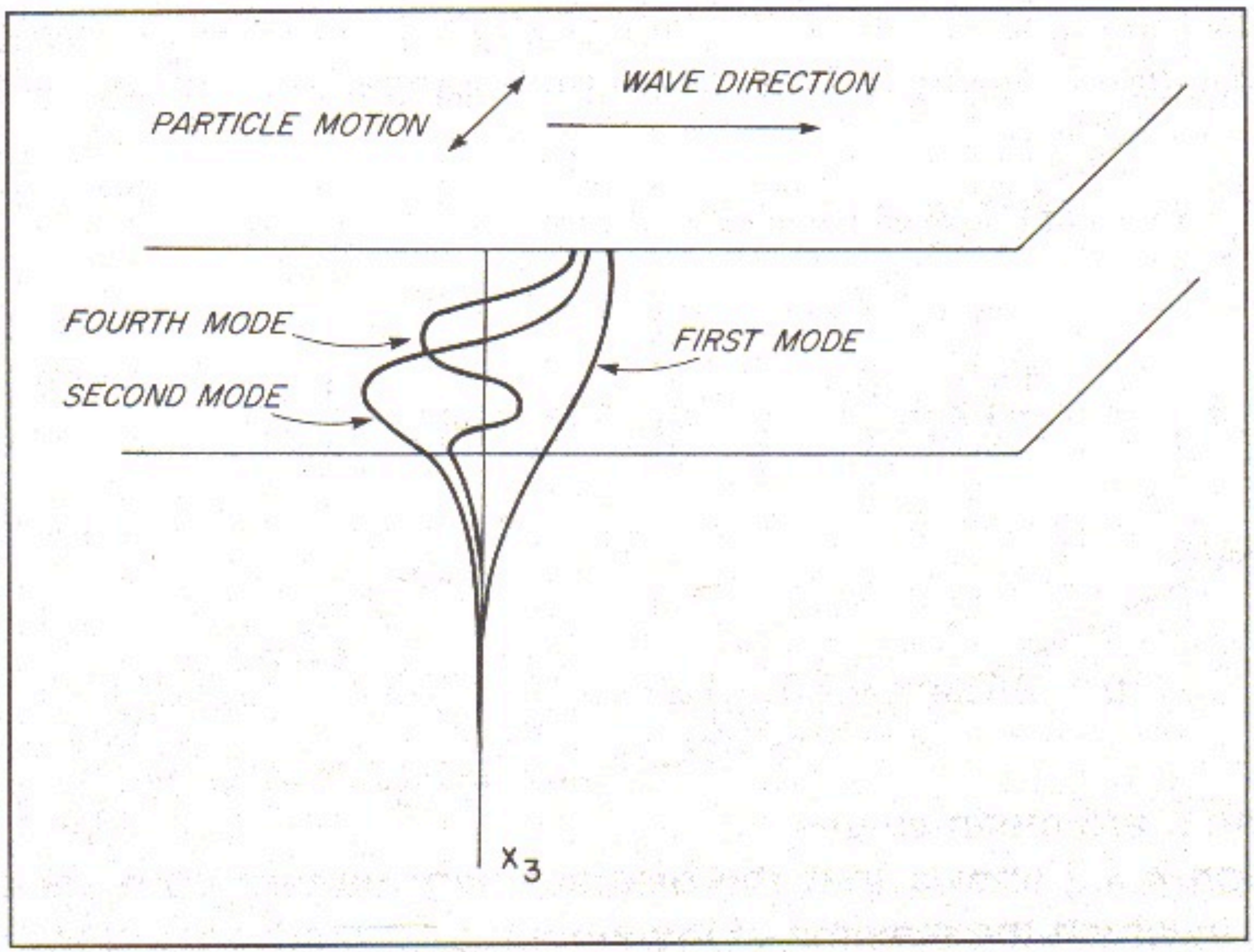
Every mode is characterized by a dispersion curve **$c=c(\omega)$**, showing the solution to the **eigenvalue** problem.

For every value of c one can calculate the **eigenfunction**, i.e. the displacement , u_y, versus depth.



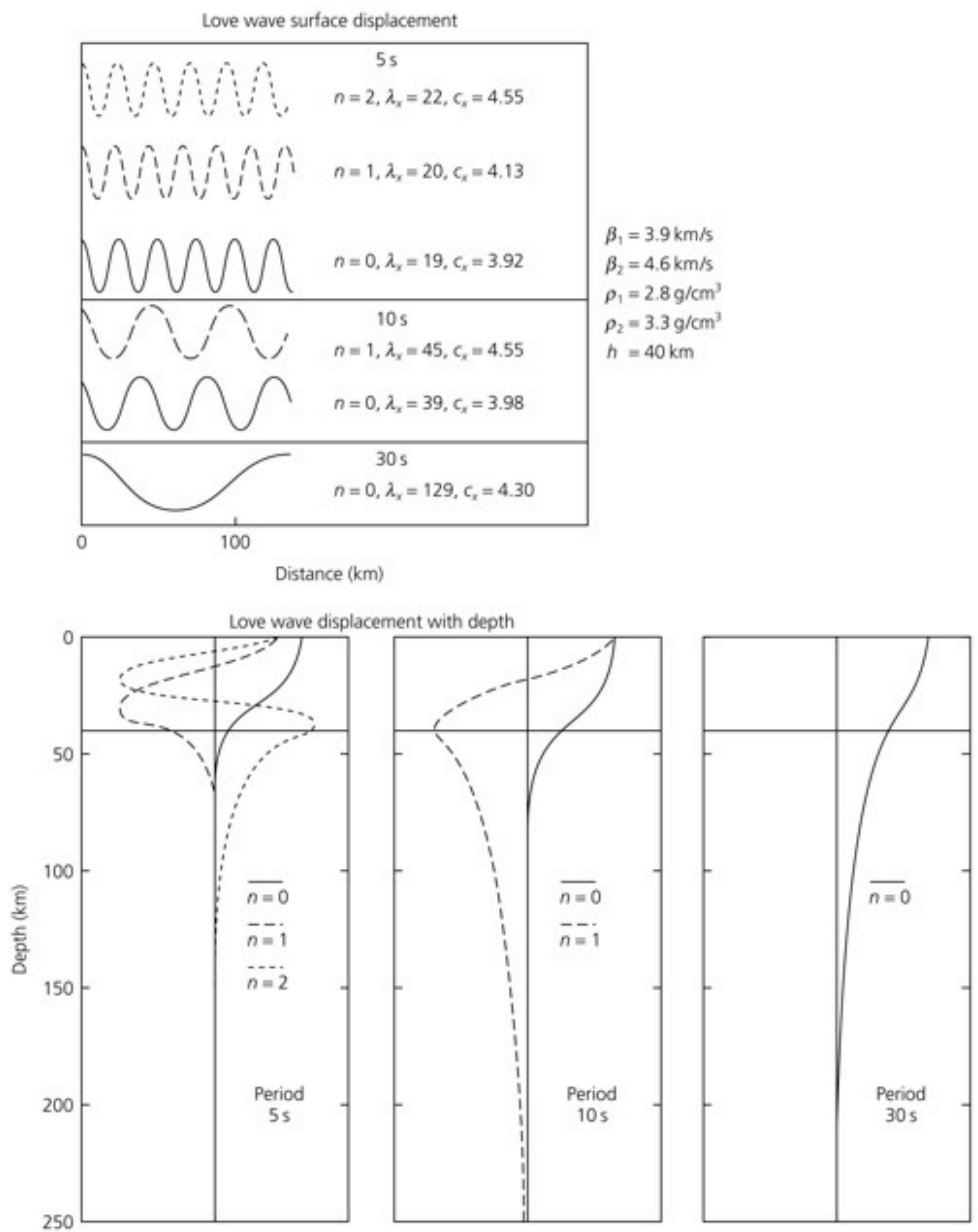
Love Waves: modes

Some modes for Love waves



Love Waves: modes

Some eigenvectors (displacement) for Love waves



Liquid layer over a half space

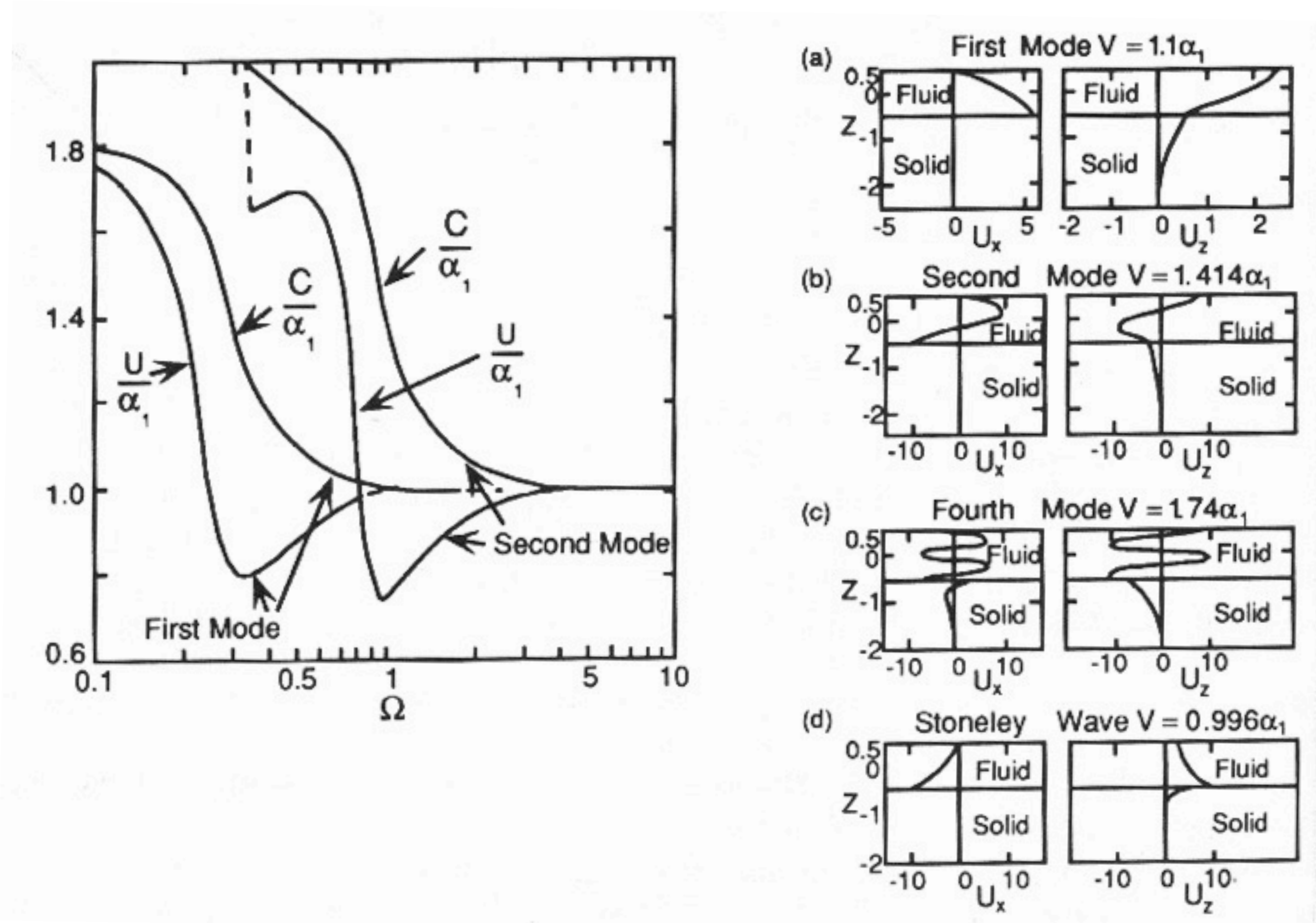
The conditions to be fulfilled are:

1. Free surface condition
2. No S-wave potential and shear stress in the liquid layer
3. Continuity of stress at the liquid-layer interface
4. Continuity of vertical component of displacement at the liquid layer interface (horizontal is free due to no viscosity in perfect liquid)

$$\tan\left(H\omega\sqrt{1/\alpha_w^2 - 1/c^2}\right) = \frac{\rho\beta^4\sqrt{c^2/\alpha_w^{\,2} - 1}}{\rho_w c^4\sqrt{1 - c^2/\alpha^2}}$$

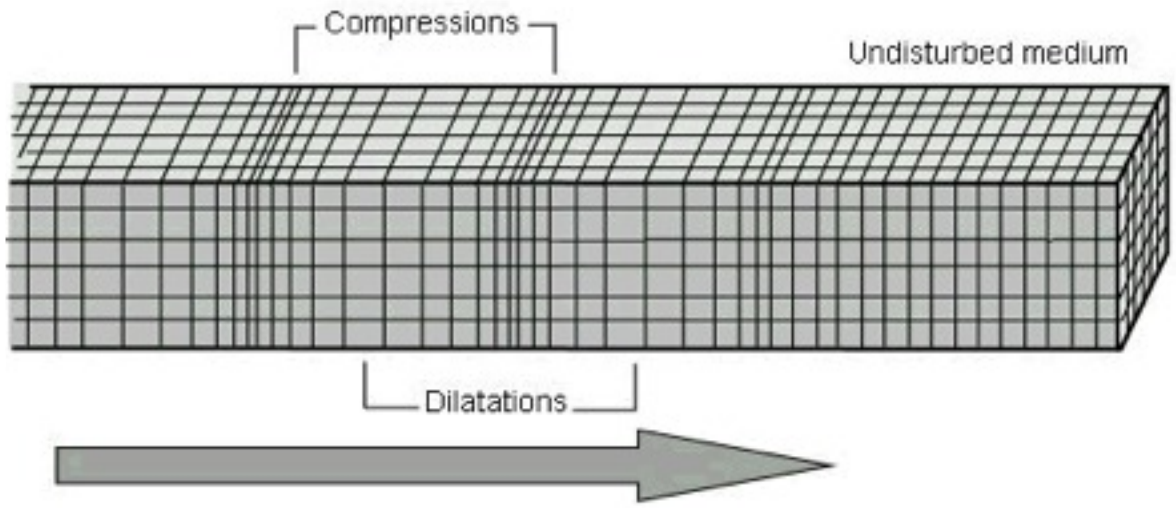
$$\left[-(2 - c^2/\beta^2)^2 + 4(1 - c^2/\alpha^2)^{1/2}(1 - c^2/\beta^2)^{1/2}\right]$$

Similar derivation for Rayleigh type motion leads to dispersive behavior

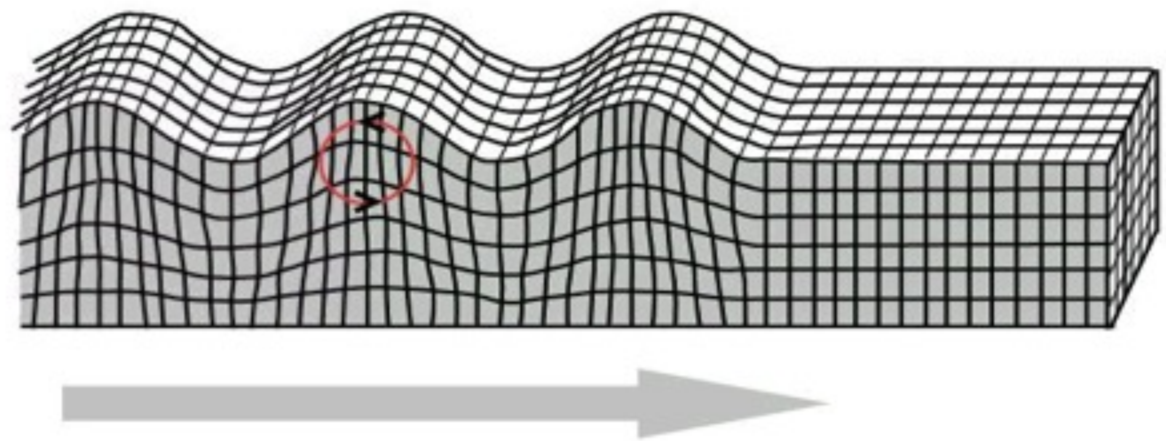


Wavefields visualization

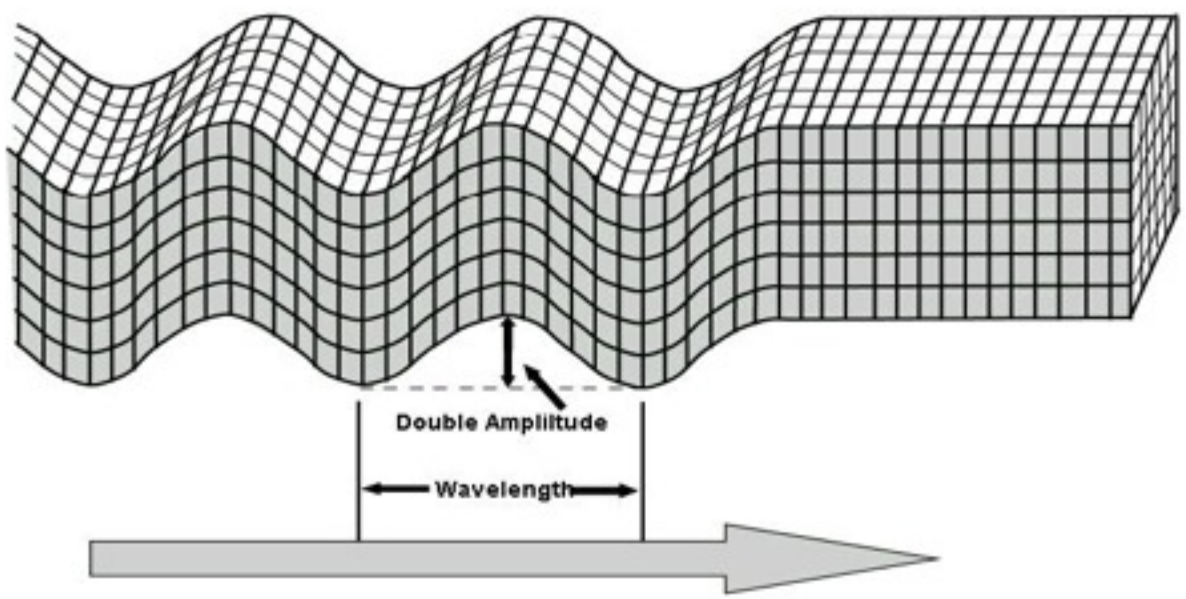
P Wave



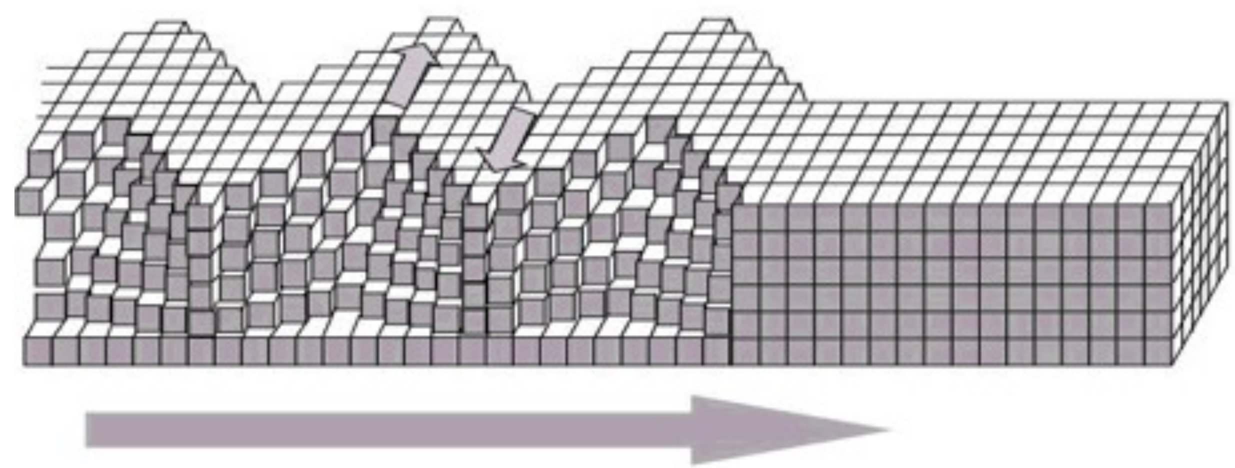
Rayleigh Wave



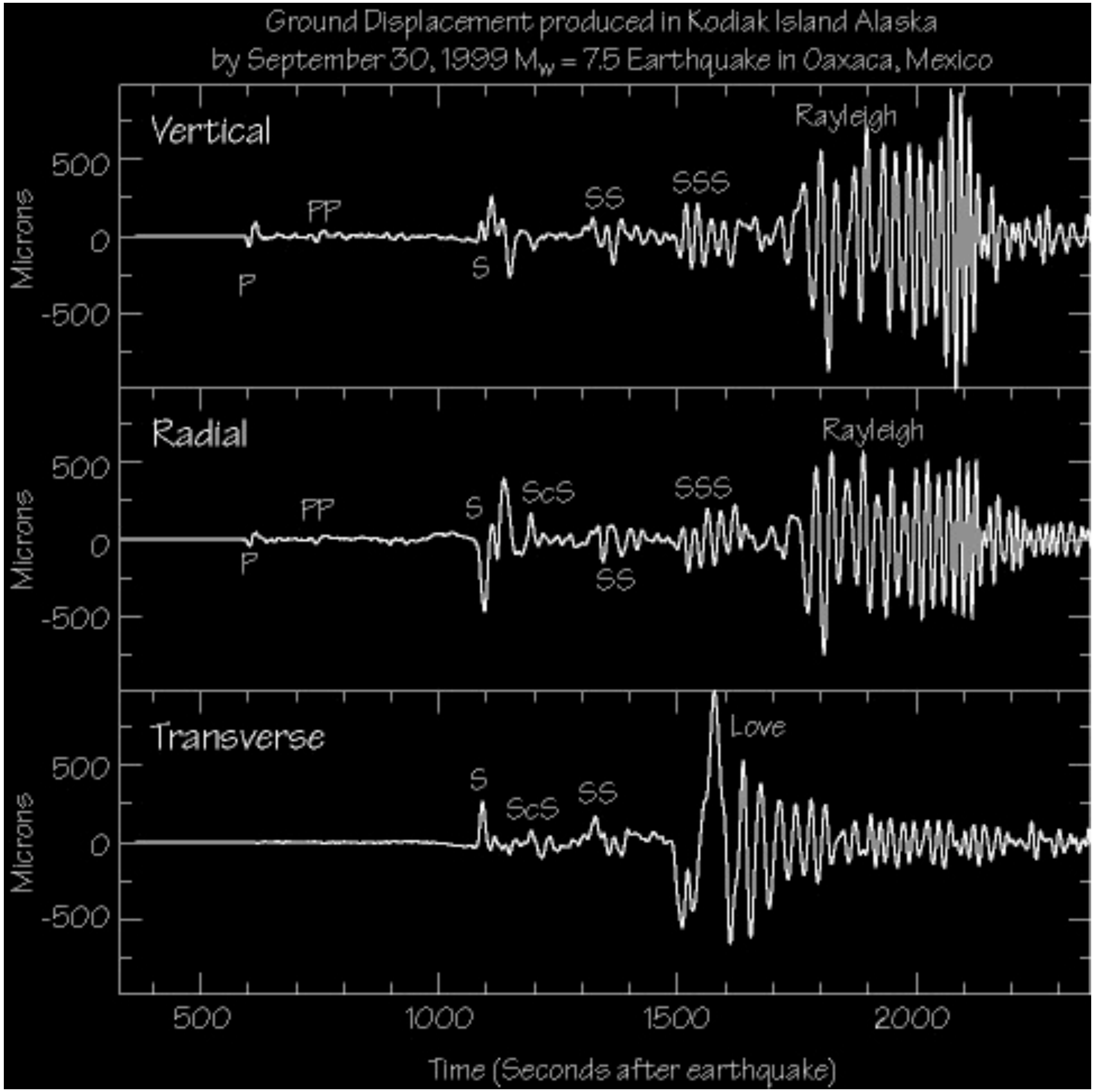
S Wave



Love Wave

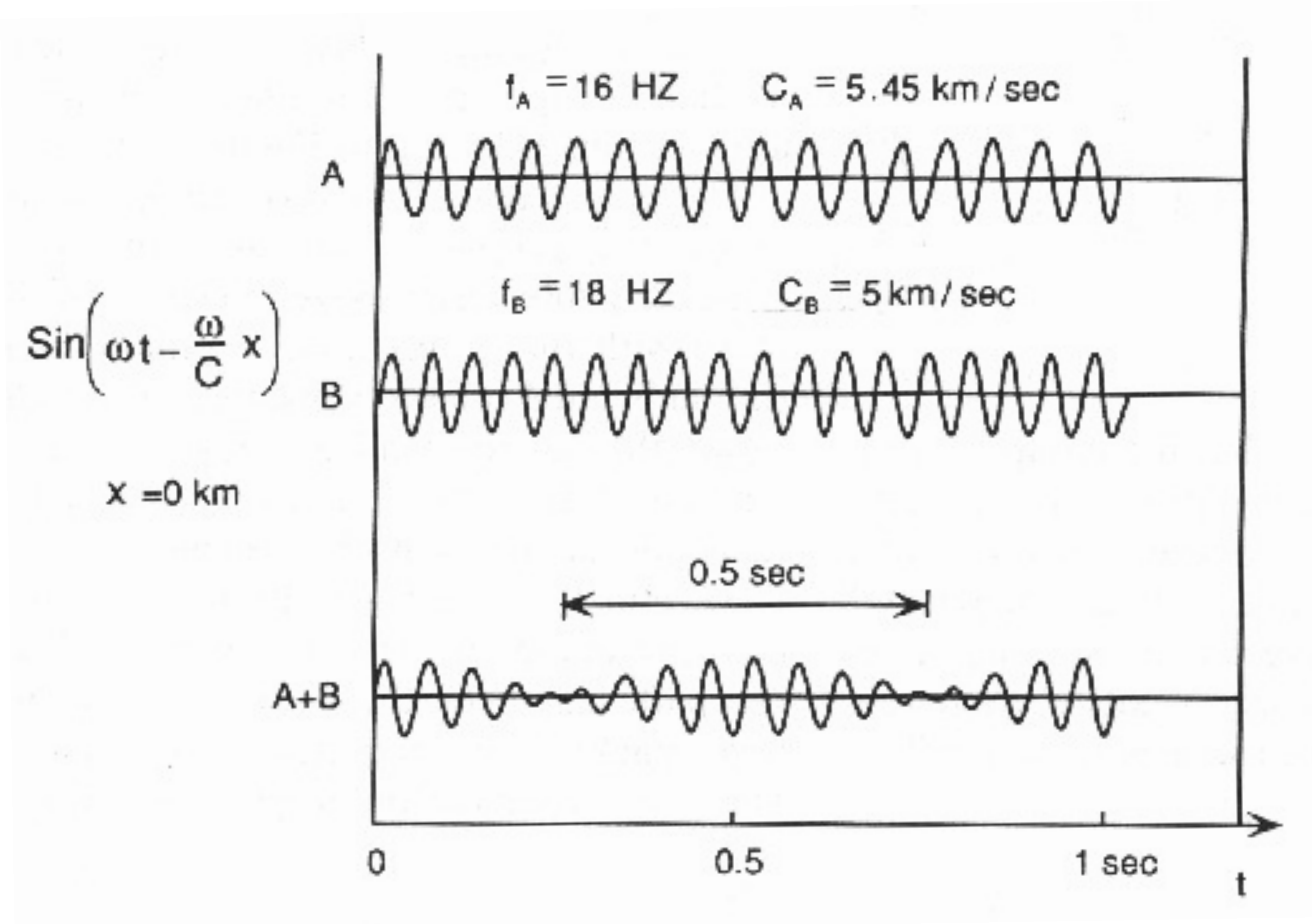


Data example - 2

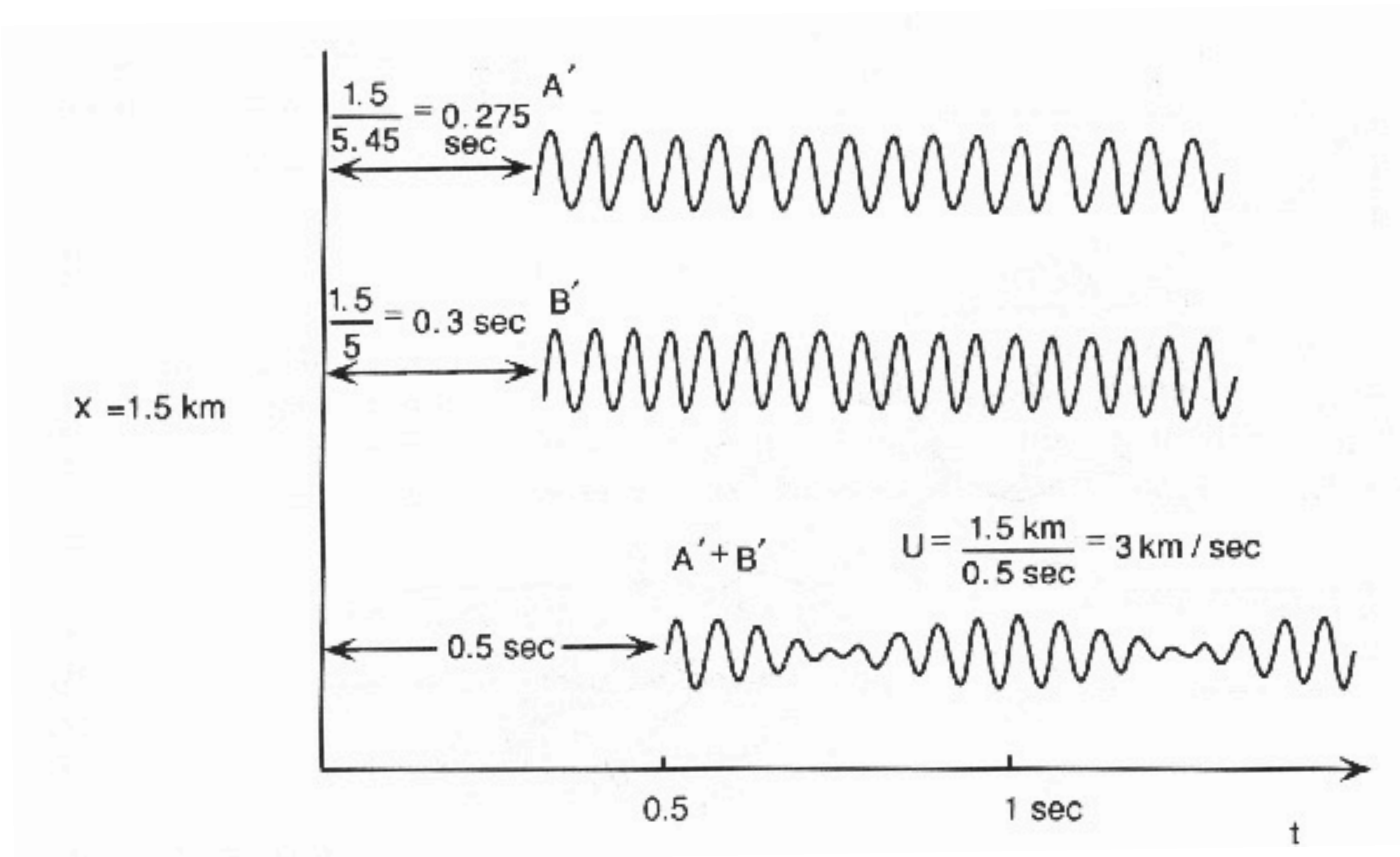


Group-velocities

Interference of two waves at two positions (1)

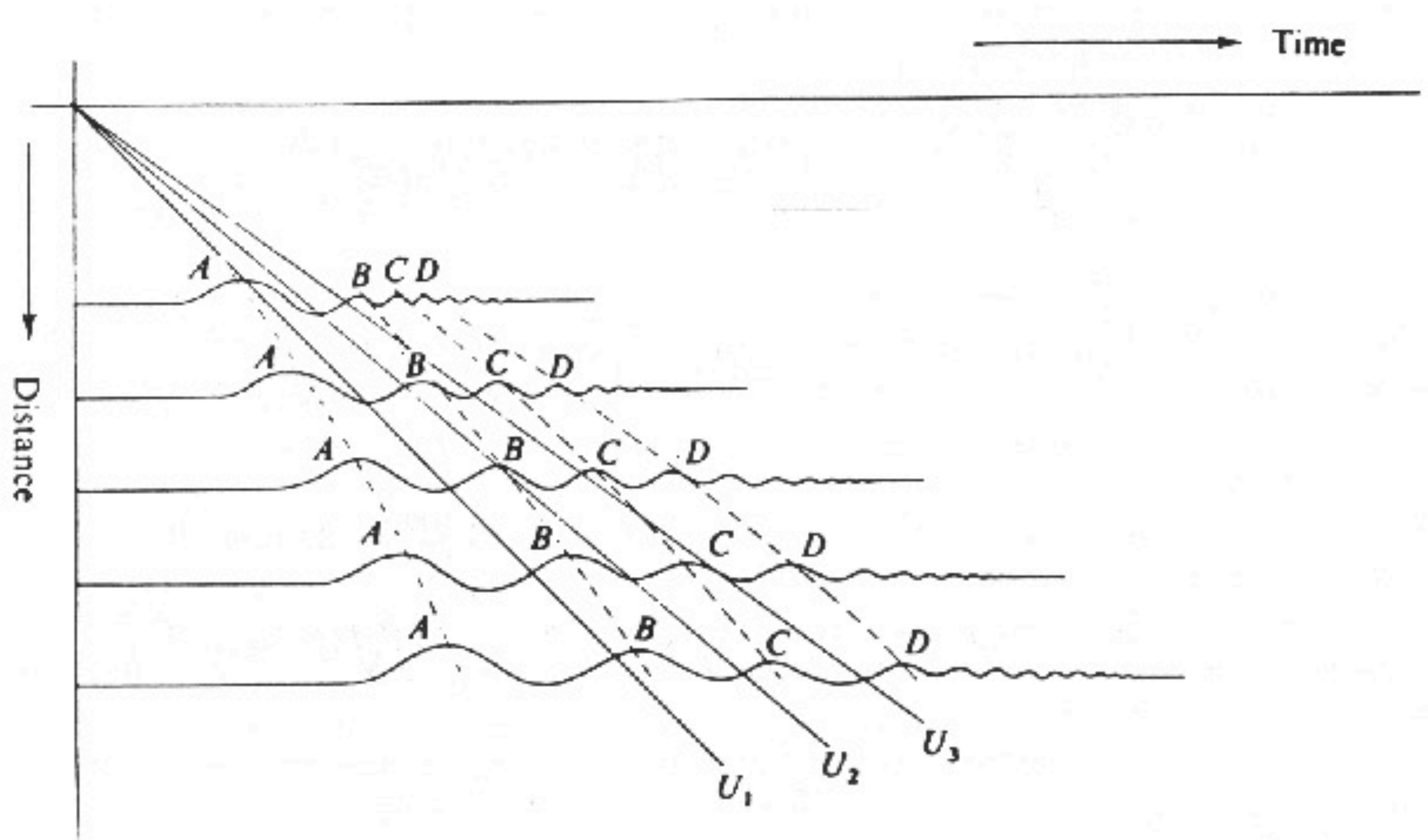


Interference of two waves at two positions (2)



Dispersion

The typical dispersive behavior of surface waves
solid – group velocities; dashed – phase velocities



Demonstration: sum two harmonic waves with slightly different angular frequencies and wavenumbers:

$$u(x, t) = \cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)$$

$$\omega_1 = \omega + \delta\omega \qquad \omega_2 = \omega - \delta\omega \qquad \omega \gg \delta\omega$$

$$k_1 = k + \delta k \qquad k_2 = k - \delta k \qquad k \gg \delta k$$

Add the two cosines:

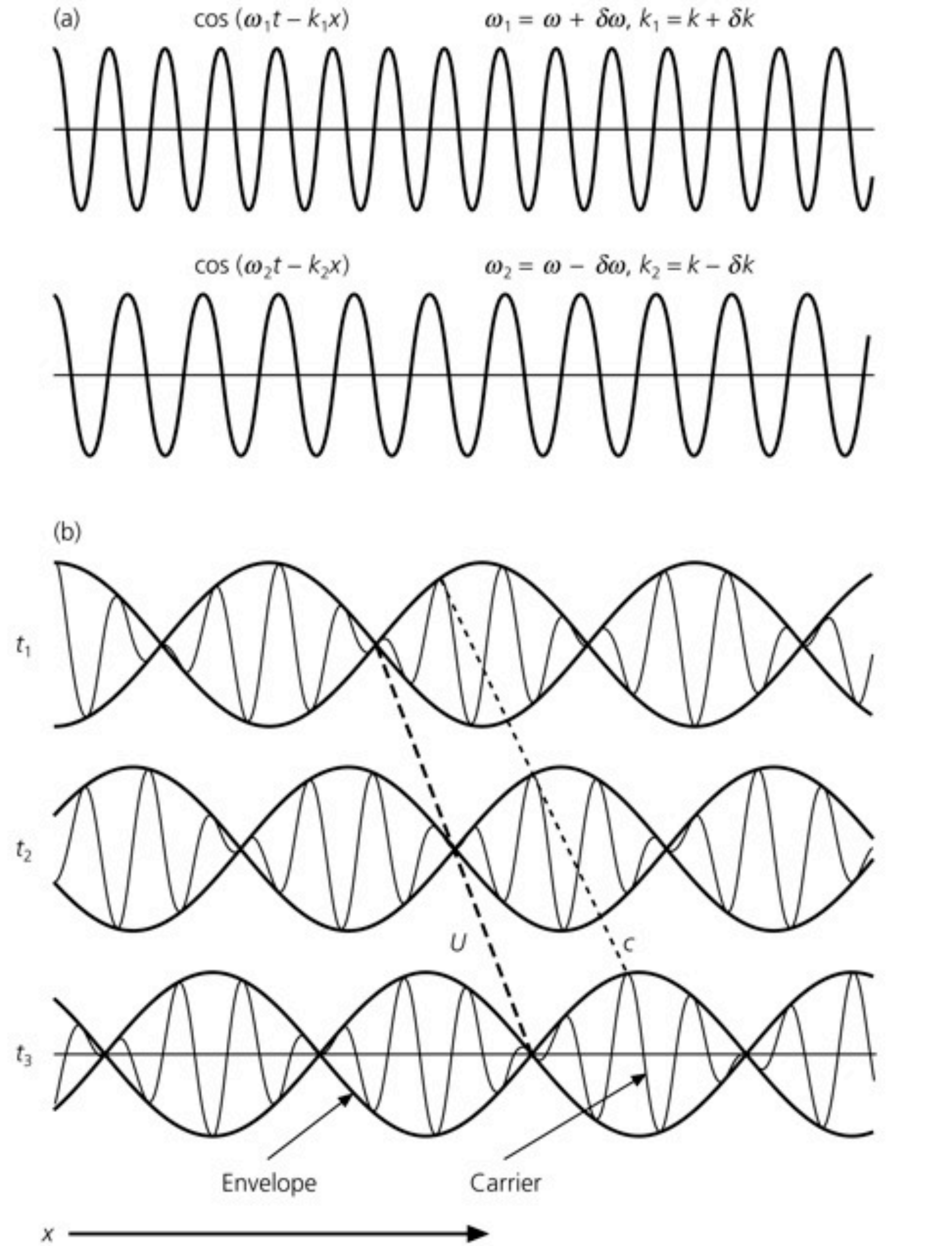
$$\begin{aligned} u(x, t) &= \cos(\omega t + \delta\omega t - kx - \delta kx) \\ &\quad + \cos(\omega t - \delta\omega t - kx + \delta kx) \\ &= 2\cos(\omega t - kx)\cos(\delta\omega t - \delta kx) \end{aligned}$$

The envelope (beat) has a *group velocity*:

$$U = \delta\omega / \delta k$$

The individual peaks move with a *phase velocity*:

$$c = \omega / k$$



Fourier domain

Fourier transform: $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt$

Inverse Fourier transform: $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t}\, d\omega$

$F(\omega) = A(\omega) e^{i\phi(\omega)}$

with a magnitude, $A(\omega) = |F(\omega)|$, and phase, $\phi(\omega)$.

So the Fourier transform represents a time series by two real functions of angular frequency: the *amplitude spectrum,* $A(\omega)$, and the *phase spectrum,* $\phi(\omega)$.

The displacements are: $u(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega) \exp i[\omega t - k(\omega)x + \phi_i(\omega)] d\omega$

The phase has two parts (propagation and initial phase): $\Phi(\omega) = \omega t - k(\omega)x + \phi_i(\omega)$

The phase velocity $c(\omega) = \omega / k(\omega)$ describes wave surfaces of constant phase (individual peaks).

To find the group velocity of energy propagation in the angular frequency band between $\omega_0 - \Delta\omega$ and $\omega_0 + \Delta\omega$, first approximate the wavenumber $k(\omega)$ by the first term of a Taylor series about ω_0:

$$k(\omega) \approx k(\omega_0) + \frac{dk}{d\omega}\bigg|_{\omega_0} (\omega - \omega_0)$$

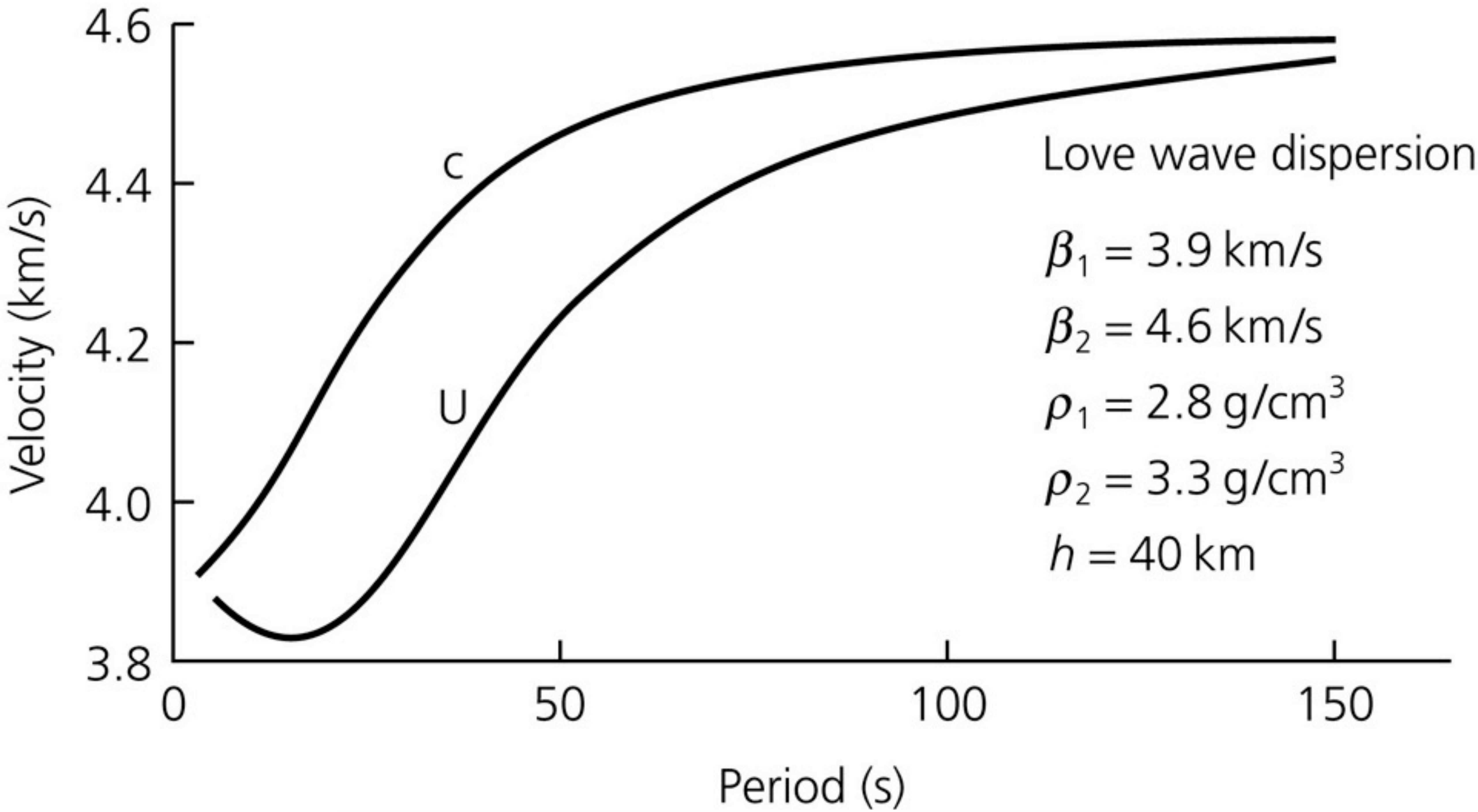
This gives: $$u(x, t) \approx \frac{1}{2\pi} \int_{\omega_0 - \Delta\omega}^{\omega_0 + \Delta\omega} A(\omega) \exp\left[i\left(\omega t - k(\omega_0)x - \frac{dk}{d\omega}\bigg|_{\omega_0}(\omega - \omega_0)x + \phi_i(\omega)\right)\right] d\omega$$

$$u(x, t) \approx \frac{1}{2\pi} \int_{\omega_0 - \Delta\omega}^{\omega_0 + \Delta\omega} A(\omega) \exp\left[i\left((\omega - \omega_0)(t - \frac{dk}{d\omega}\bigg|_{\omega_0} x) + (\omega_0 t - k(\omega_0)x) + \phi_i(\omega)\right)\right] d\omega$$

Compare to the simple situation of two cosine waves:

$$u(x, t) = 2\cos(\omega t - kx)\cos(\delta\omega t - \delta kx)$$

Similar to the cosine waves, the group velocity is defined as $U(\omega) = \dfrac{d\omega}{dk}$

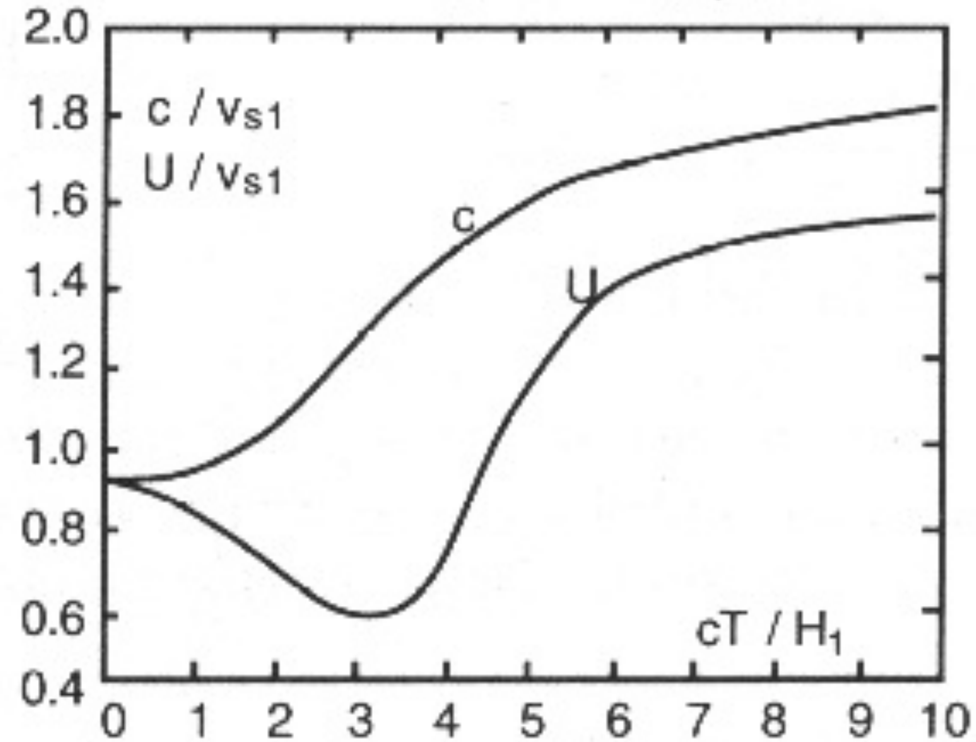


$$U = \frac{d\omega}{dk} = \frac{d(ck)}{dk} = c + k\frac{dc}{dk} = c - \lambda \frac{dc}{d\lambda}$$

Dispersion

Fundamental Mode Rayleigh dispersion curve for
a layer over a half space.

Layer	V_p	V_s	ρ	H
1	$1.732 v_{s1}$	v_{s1}	ρ_1	H_1
2	$3.873 v_{s1}$	$2.236 v_{s1}$	ρ_1	∞



Dispersion...

The group velocity itself is usually a function of the wave's frequency. This results in group velocity dispersion (GVD),that is often quantified as the group delay dispersion parameter : If D is < 0, the medium is said to have **positive dispersion**. If D is > 0, the medium has **negative dispersion**.

$$D = \frac{dv_g}{d\omega}$$

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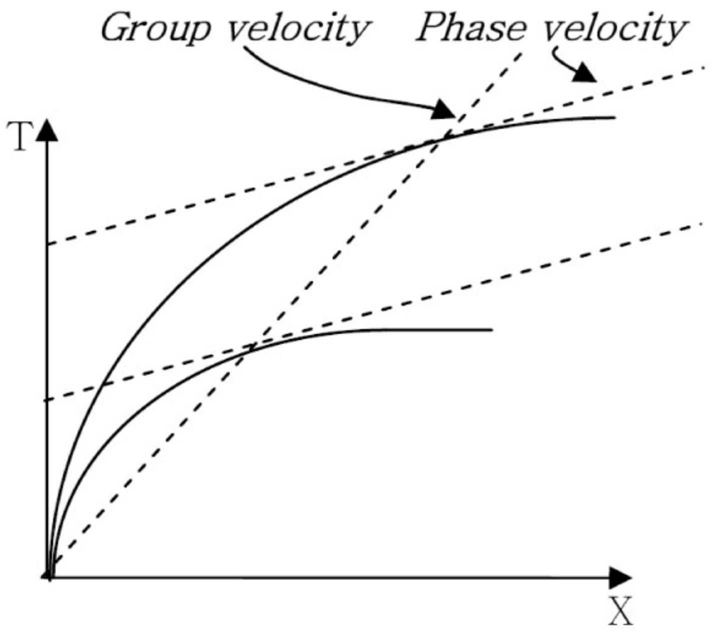

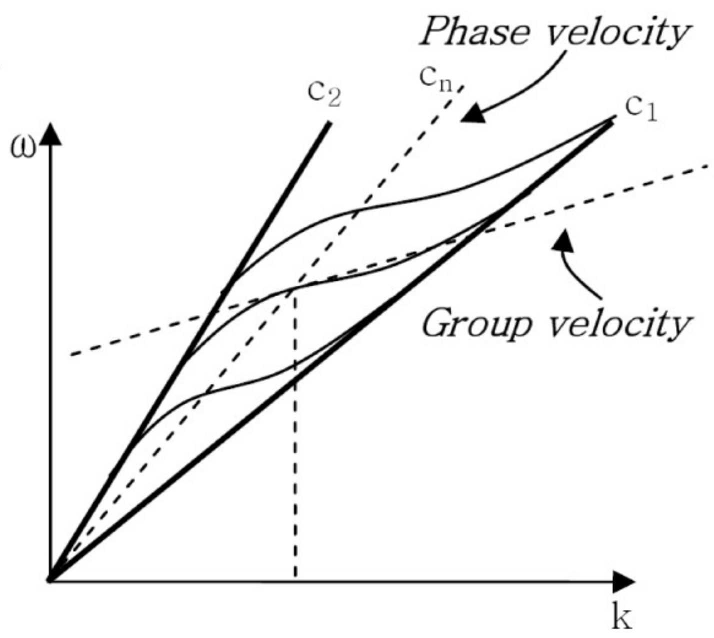

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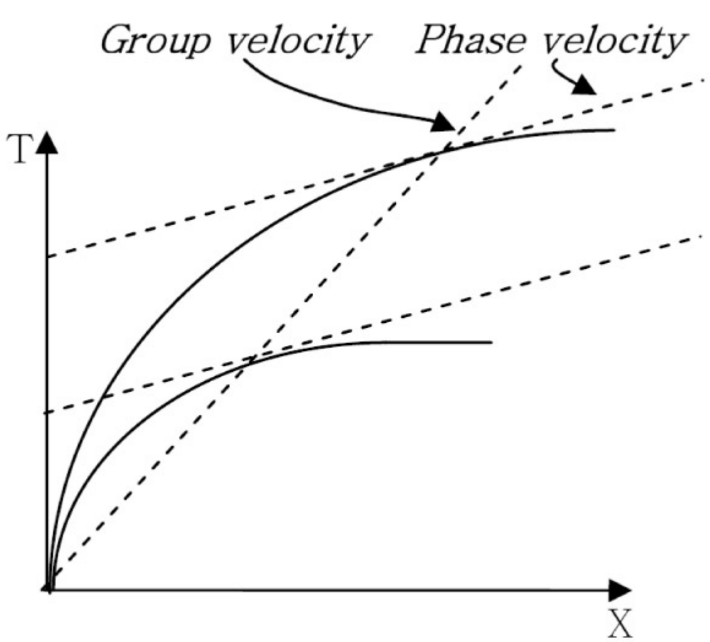
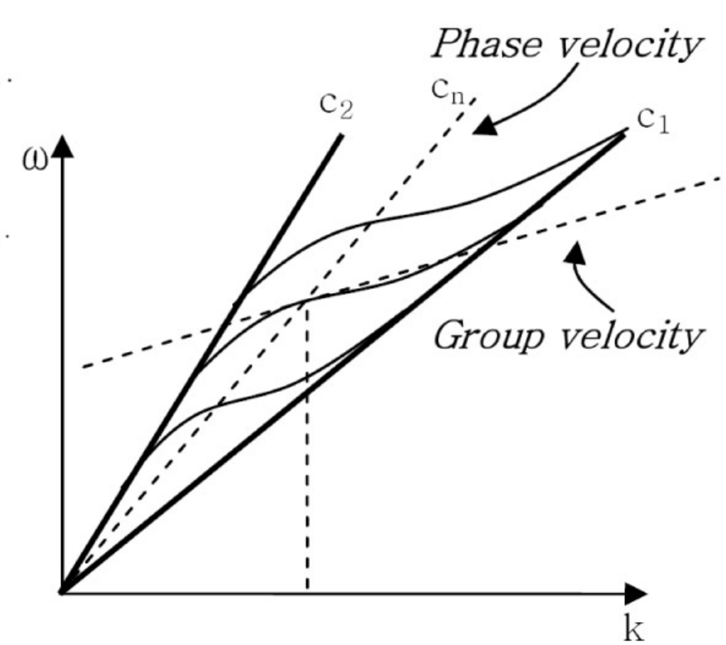
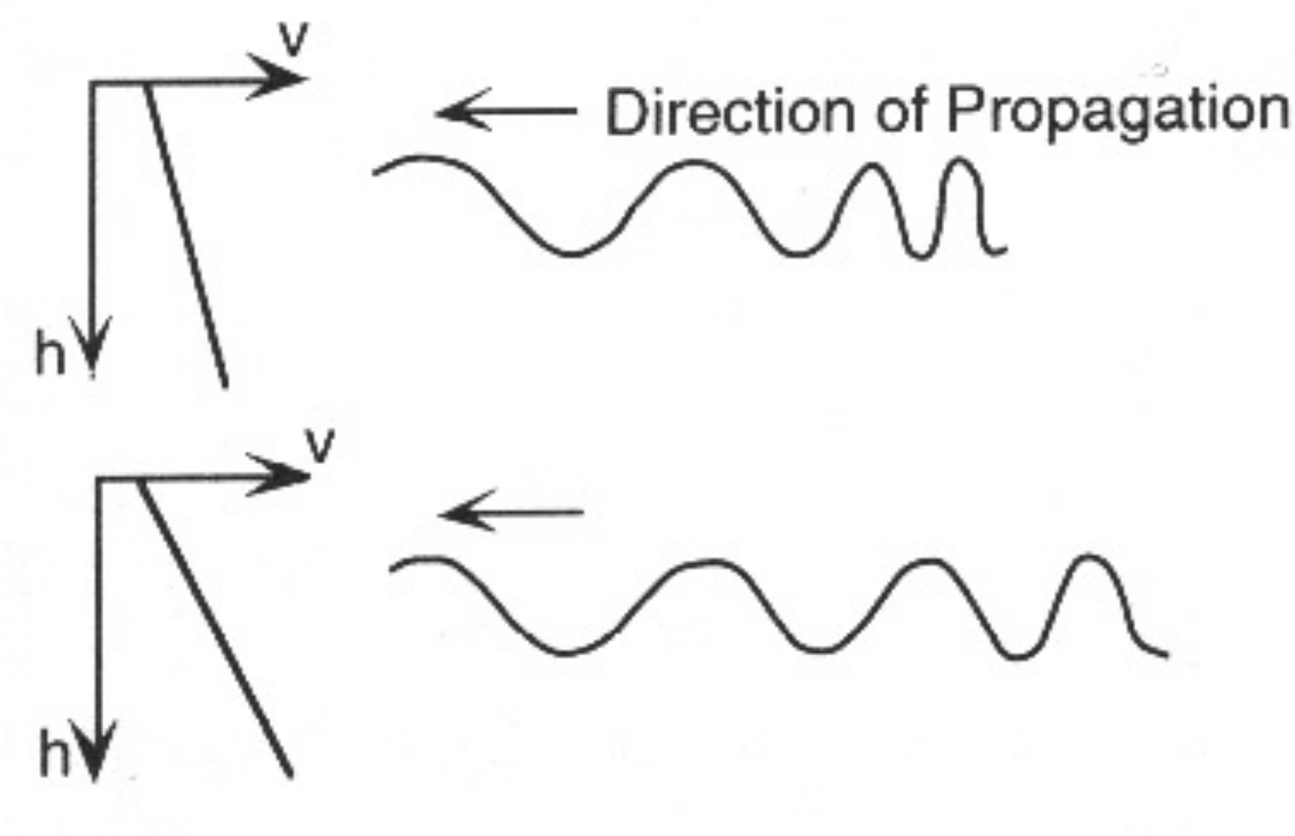




Airy Phase –
wave that arises if the phase and the change in group velocity are stationary and gives the highest amplitude in terms of group velocity and are prominent on the seismogram.

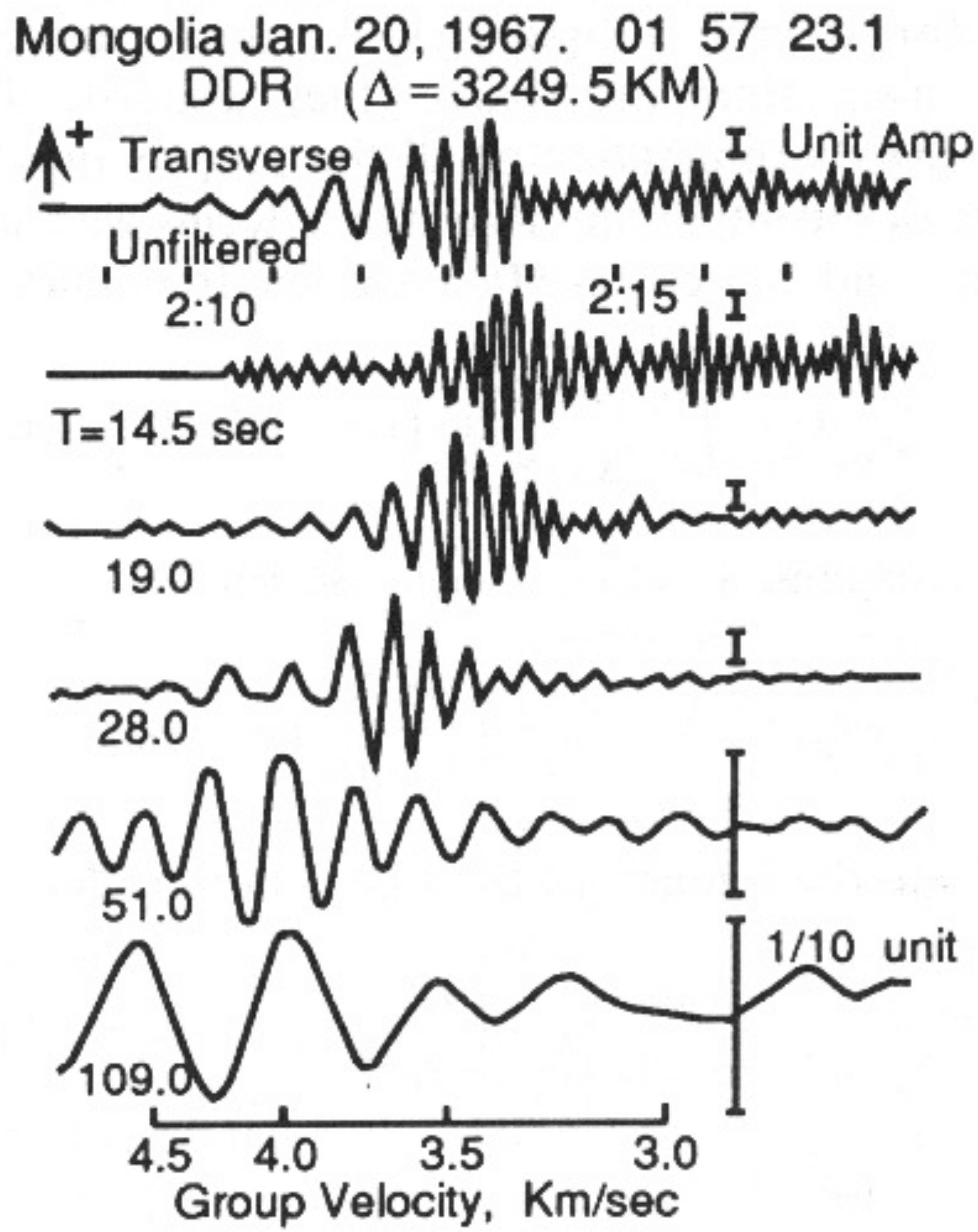
Dispersion



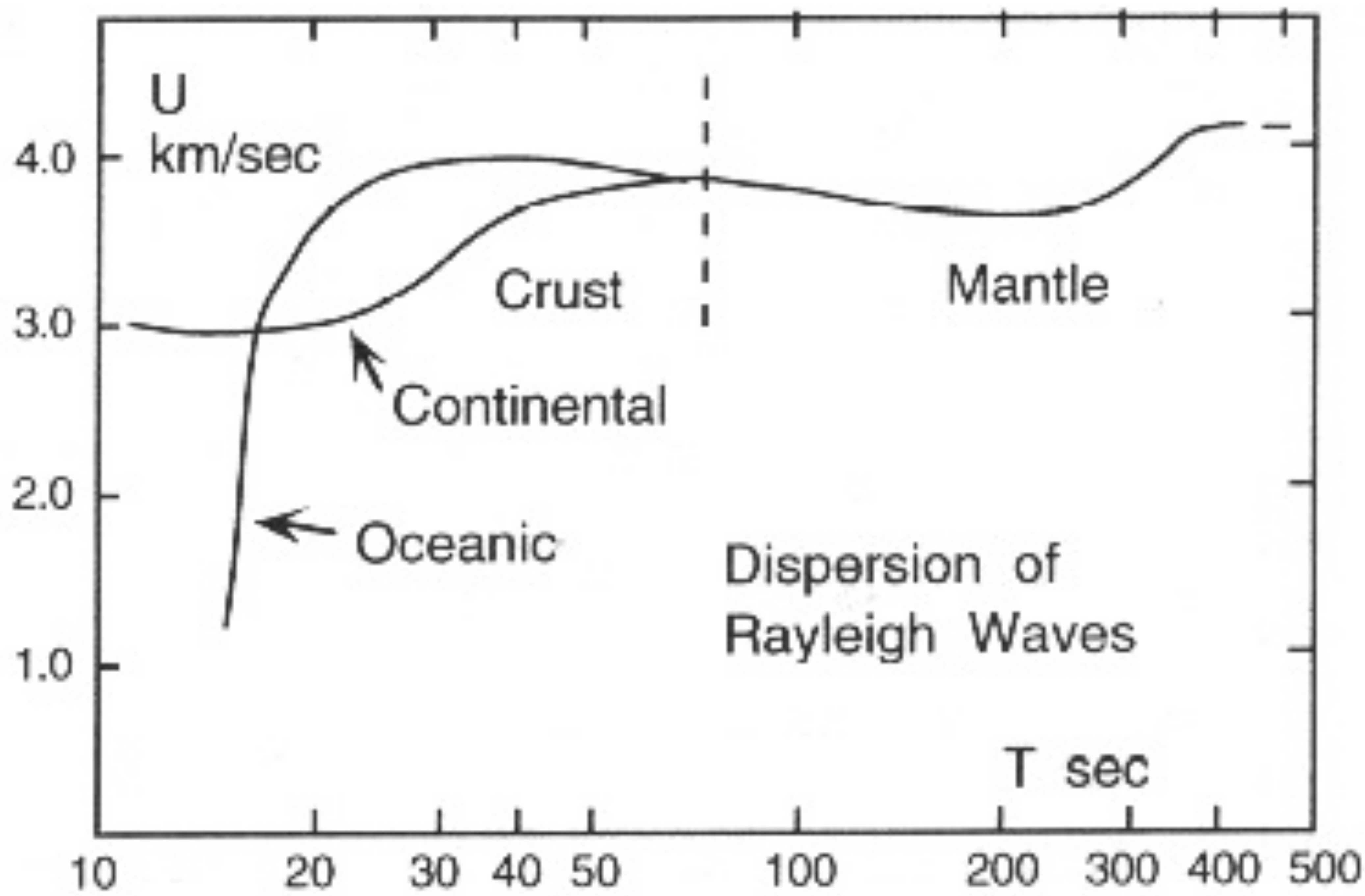
Stronger gradients cause greater dispersion

Wave Packets

Seismograms of a Love wave train filtered with different central periods. Each narrowband trace has the appearance of a wave packet arriving at different times.



Observed Group Velocities (T<80s)



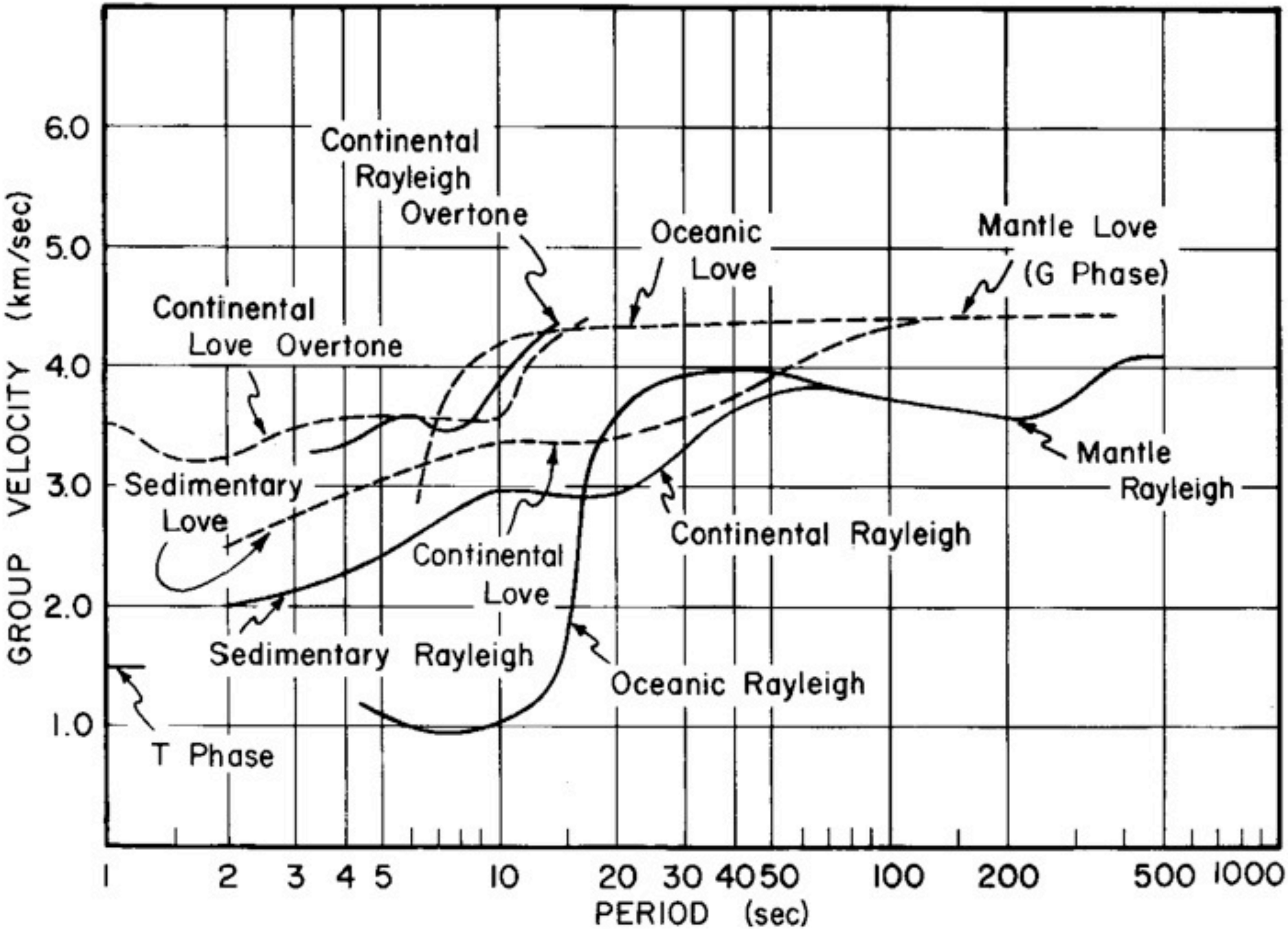


Figure 1.2.1b Composite of dispersion curves for surface waves.

Measuring group velocity

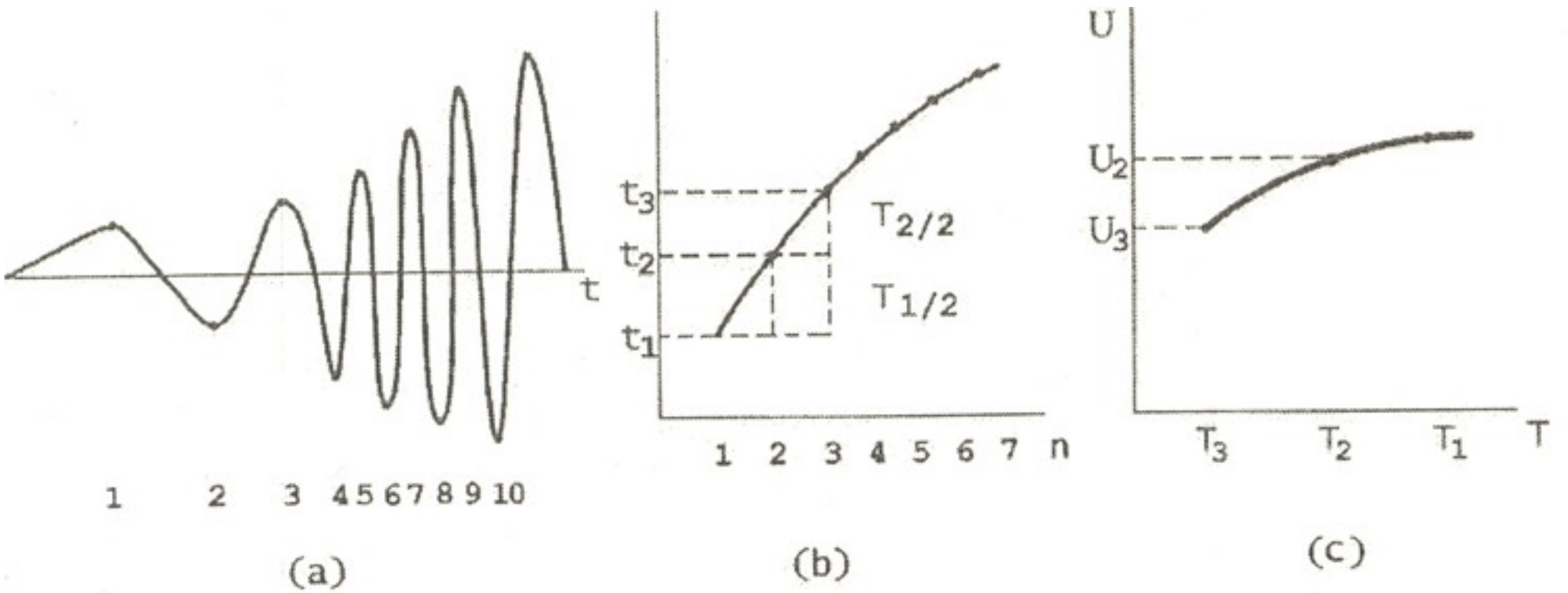
One station method

1. Directly measure the arrival time of the peaks and troughs on one seismogram
2. Narrow filtered the seismogram, measure the arrival of the peak of the wave packet (more accurate)

$$U(\omega) = \frac{x}{t}$$

Need know the origin time and the location of the earthquake source

Determination of group velocities at one station



Measuring group velocity

Two stations method

If two stations are located on the same great circle path, group velocity can be obtained by measuring the difference in arrival times of filtered wave packets.

$$U(\omega) = \frac{\Delta x}{\Delta t}$$

Distance between two stations (Δx)

Different of arrival times at the two stations (Δt)

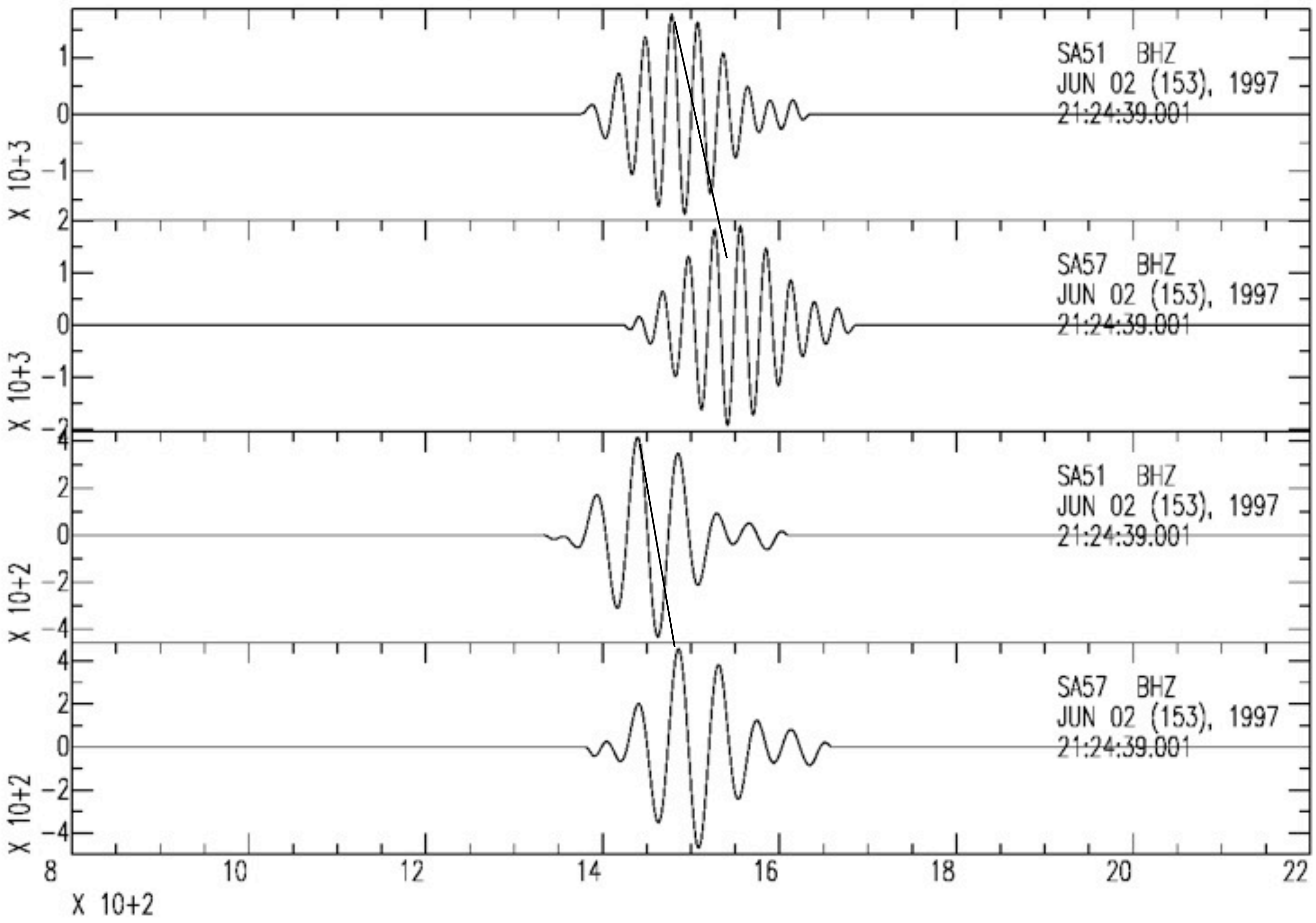
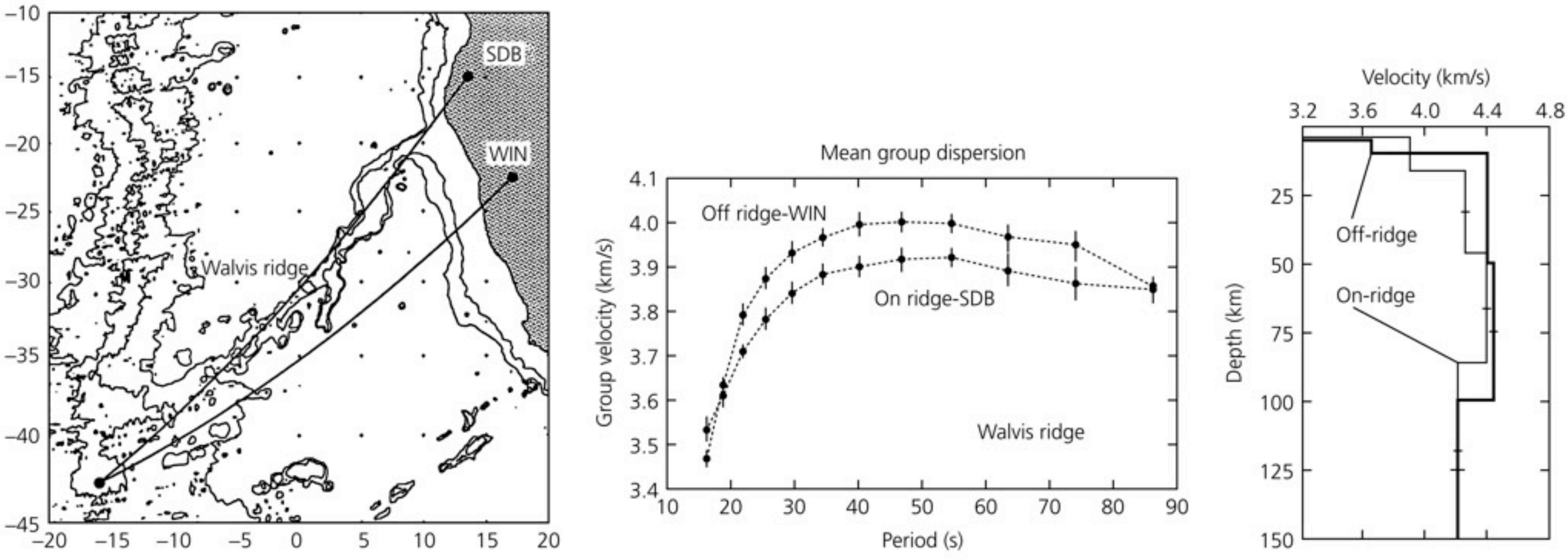
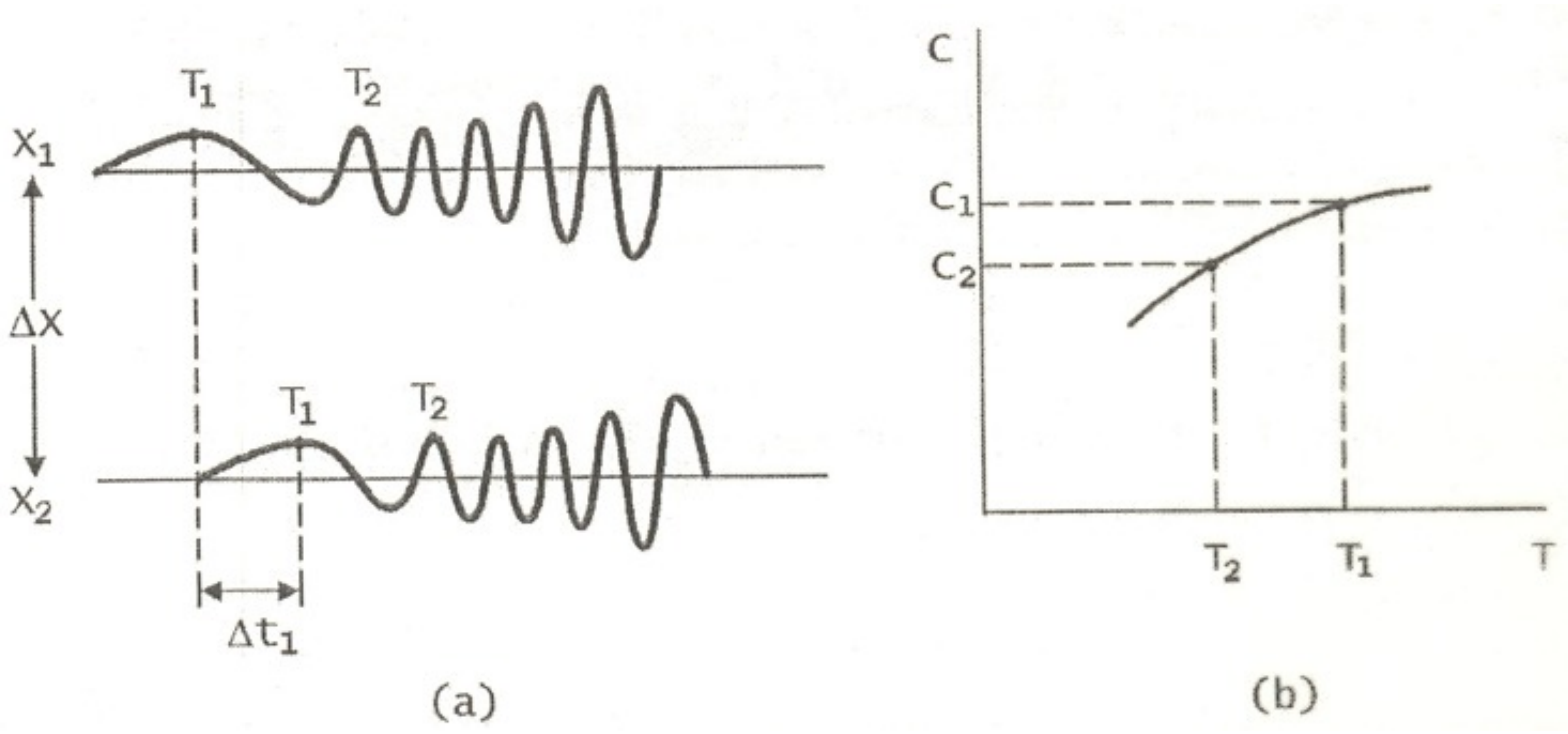

SA51 BHZ
JUN 02 (153), 1997
21:24:39.001
SA57 BHZ
JUN 02 (153), 1997
21:24:39.001
SA51 BHZ
JUN 02 (153), 1997
21:24:39.001
SA57 BHZ
JUN 02 (153), 1997
21:24:39.001
X 10+3
X 10+3
X 10+2
X 10+2
X 10+2

Figure 2.8-5: Rayleigh wave group velocity study of the Walvis ridge.



Directly measured at two stations



Measuring phase velocity

Measured by taking Fourier transform and obtaining phase spectrum

A surface wave can be represented:

$$u(x,t) = \frac{1}{\pi}\int_0^\infty A(\omega, x)\cos(\omega t - \frac{\omega}{c(\omega)}x + \phi_0(x))d\omega$$

$$\phi(\omega) = \omega t - \frac{\omega}{c(\omega)}x + \phi_0(\omega) + 2\pi N$$

One-station method

$$\phi_1(\omega) = \omega t_1 - \frac{\omega}{c(\omega)} x_1 + \phi_0(\omega) + 2\pi N$$

One-station method

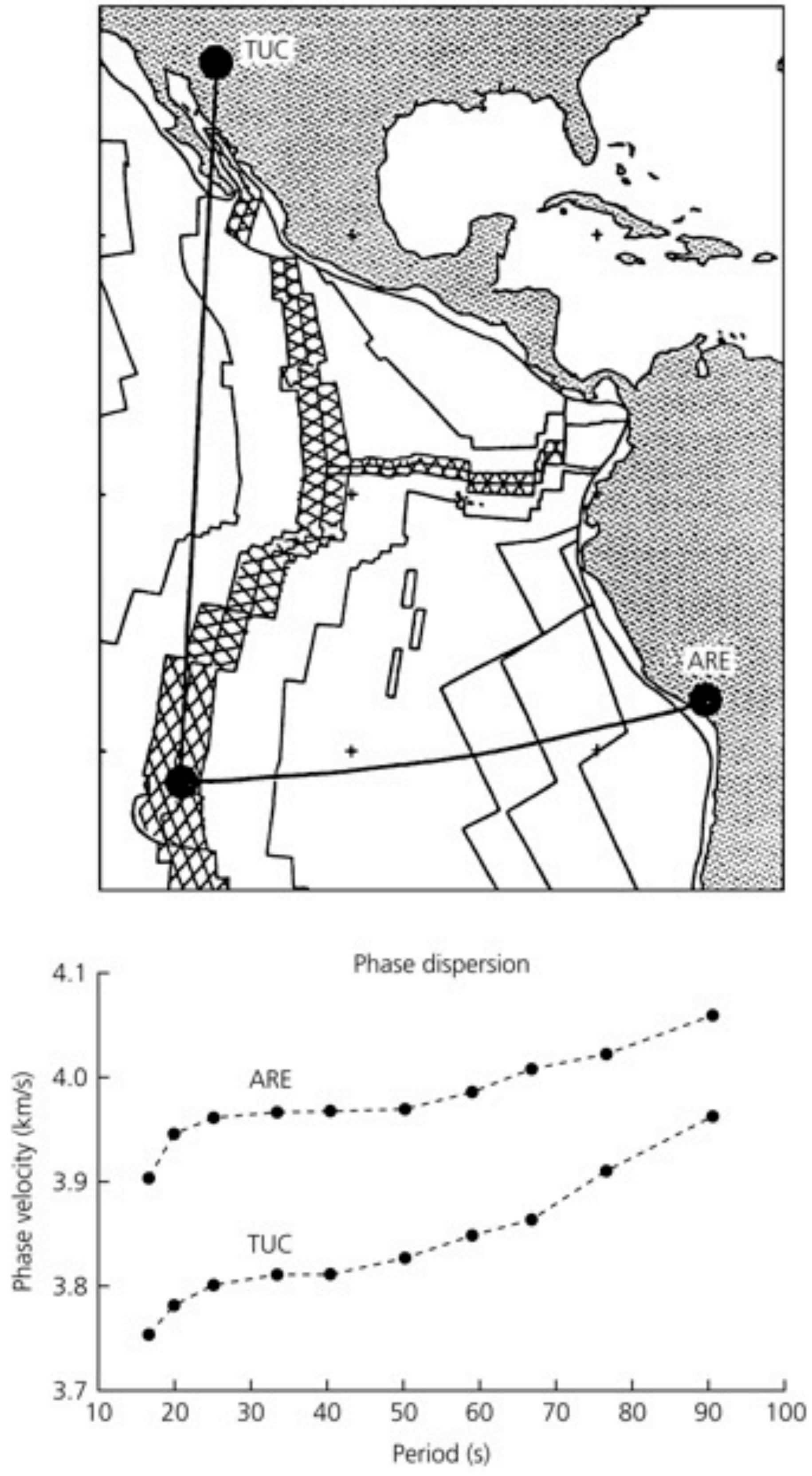
$$\phi_1(\omega) = \omega t_1 - \frac{\omega}{c(\omega)} x_1 + \phi_0(\omega) + 2\pi N$$

Need know the initial phase ϕ_0

N can be determined by by allowing c(ω) for the longest period converge to the global average

c measures

Figure 2.8-6: Example of Rayleigh wave phase velocities for ocean lithosphere.



On a seismogram recorded at a distance x from the earthquake at time t after the earthquake, the phase has three terms:

$$\Phi(\omega) = [\omega t - k(\omega)x] + \phi_i(\omega) + 2n\pi$$

$$= [\omega t - \omega x / c(\omega)] + \phi_i(\omega) + 2n\pi$$

$\omega t - k(\omega)x$ is the phase due to the propagation of the wave in time and space.

$\phi_i(\omega)$ includes the initial phase at the earthquake and any phase shift introduced by the seismometer.

$2n\pi$ reflects the periodicity of the complex exponential, because adding an integral multiple of 2π to the argument yields the same value.

Two station method:

$$\Phi_1(\omega) = \omega t_1 - \omega x_1/c(\omega) + \phi_i(\omega) + 2n\pi$$

$$\Phi_2(\omega) = \omega t_2 - \omega x_2/c(\omega) + \phi_i(\omega) + 2m\pi$$

Take the difference $\Phi_{21} = \Phi_2 - \Phi_1$, and solve for c:

$$c(\omega) = \omega(x_2 - x_1)/[\omega(t_2 - t_1) + 2(m - n)\pi - \Phi_{21}(\omega)].$$

The $2(m - n)\pi$ term is found empirically by ensuring that the phase velocity at long periods is reasonable.

Single station method:
Predict the phase at the earthquake from its focal mechanism.
If $\phi_i(\omega)$ is known, c is:

$$c(\omega) = \omega x/[\omega t + \phi_i(\omega) + 2n\pi - \Phi(\omega)]$$

Figure 2.8-6: Example of Rayleigh wave phase velocities for ocean lithosphere.

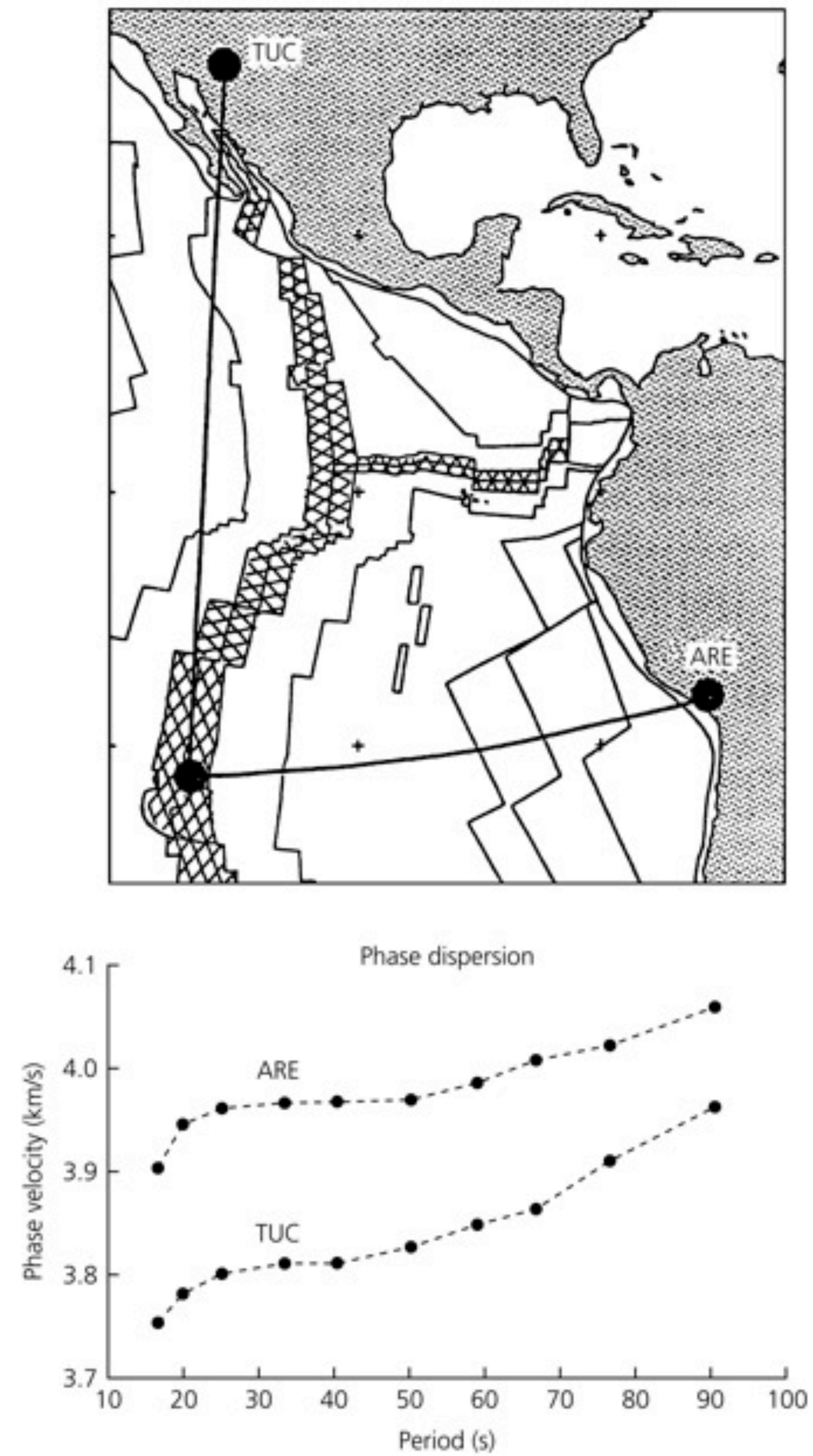


Figure 2.8-7: Rayleigh wave phase velocity dispersion as a function of oceanic plate age.