Theoretical Seismology

Astrophysics and Cosmology and Earth and Environmental Physics

Seismic sources 1: faulting

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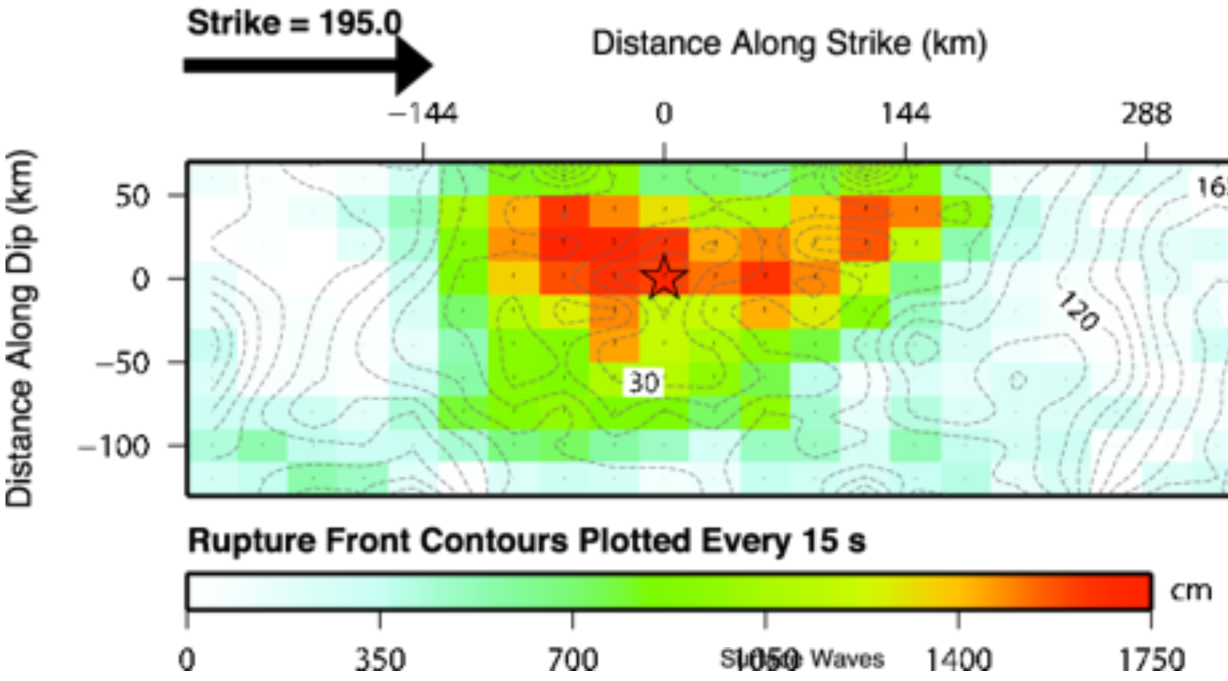
Seismic sources - 1

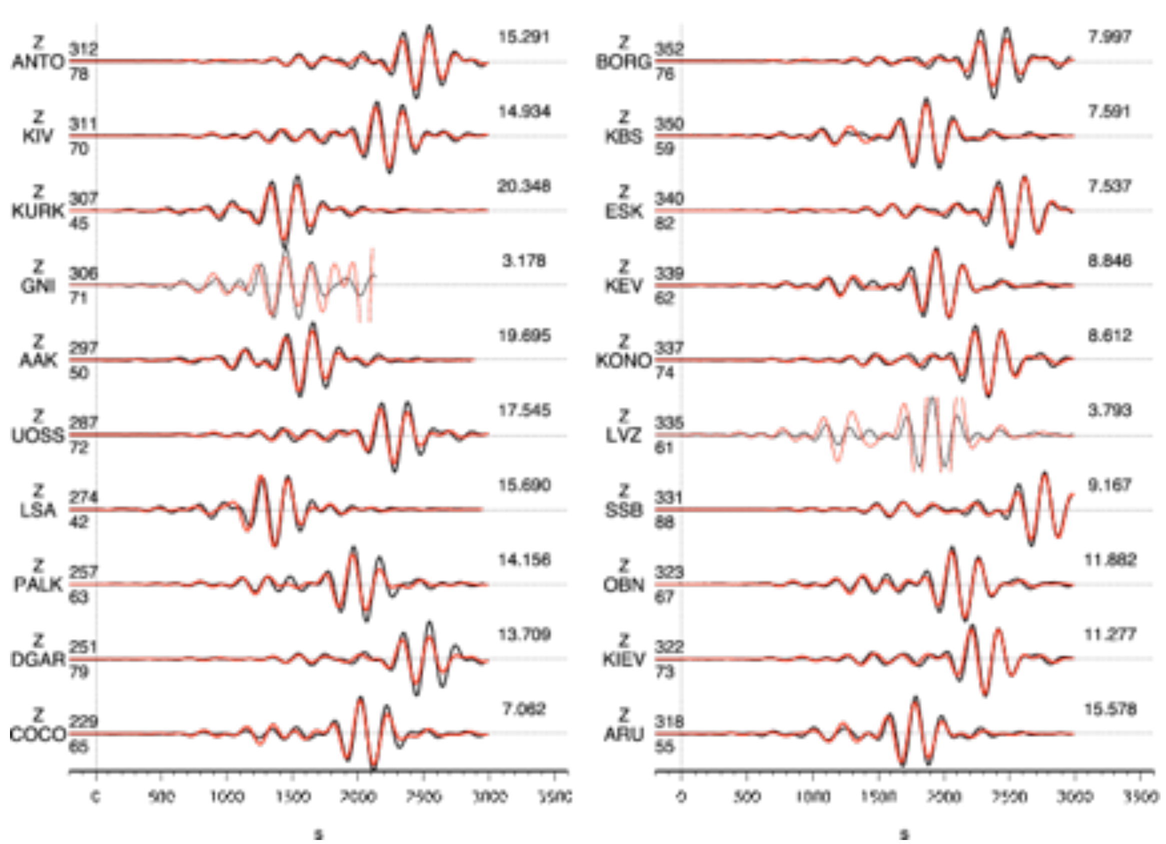
Basic Dynamics

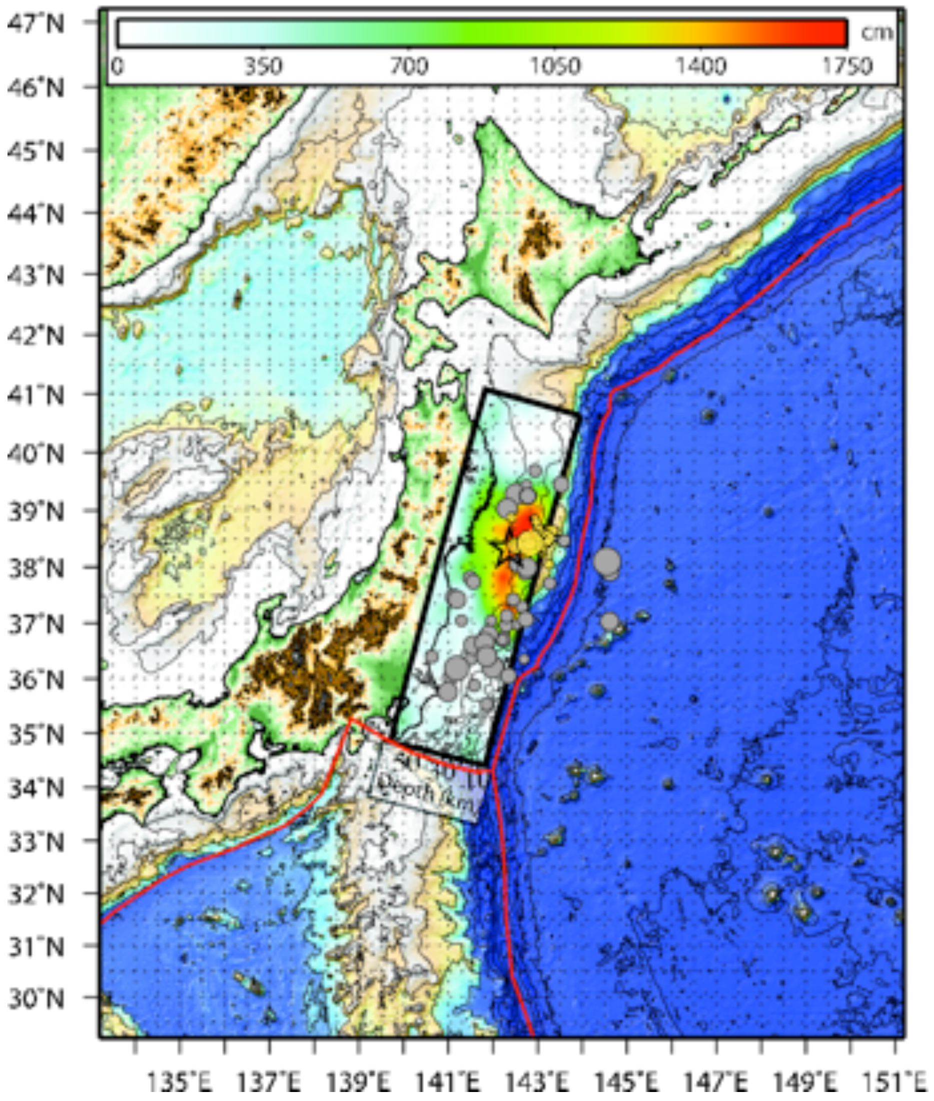
- **rupture process**
  - fault geometry
- **brittle deformation**
  - stress states
- **stresses and faulting**
  - stress cycle (stick-slip)

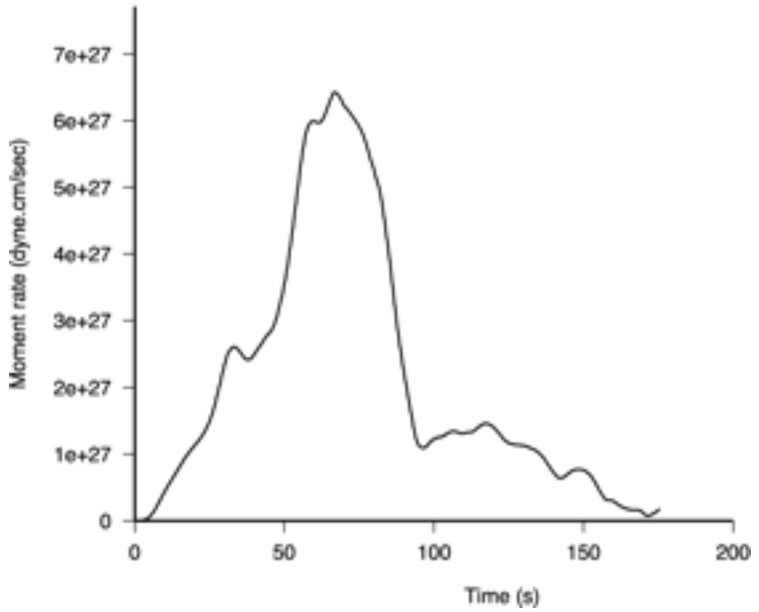
USGS - Finite fault model - Tohoku

Cross-section of slip distribution. The strike direction of the fault plane is indicated by the black arrow and the hypocenter location is denoted by the red star. The slip amplitude are showed in color and motion direction of the hanging wall relative to the footwall is indicated by black arrows. Contours show the rupture initiation time in seconds.

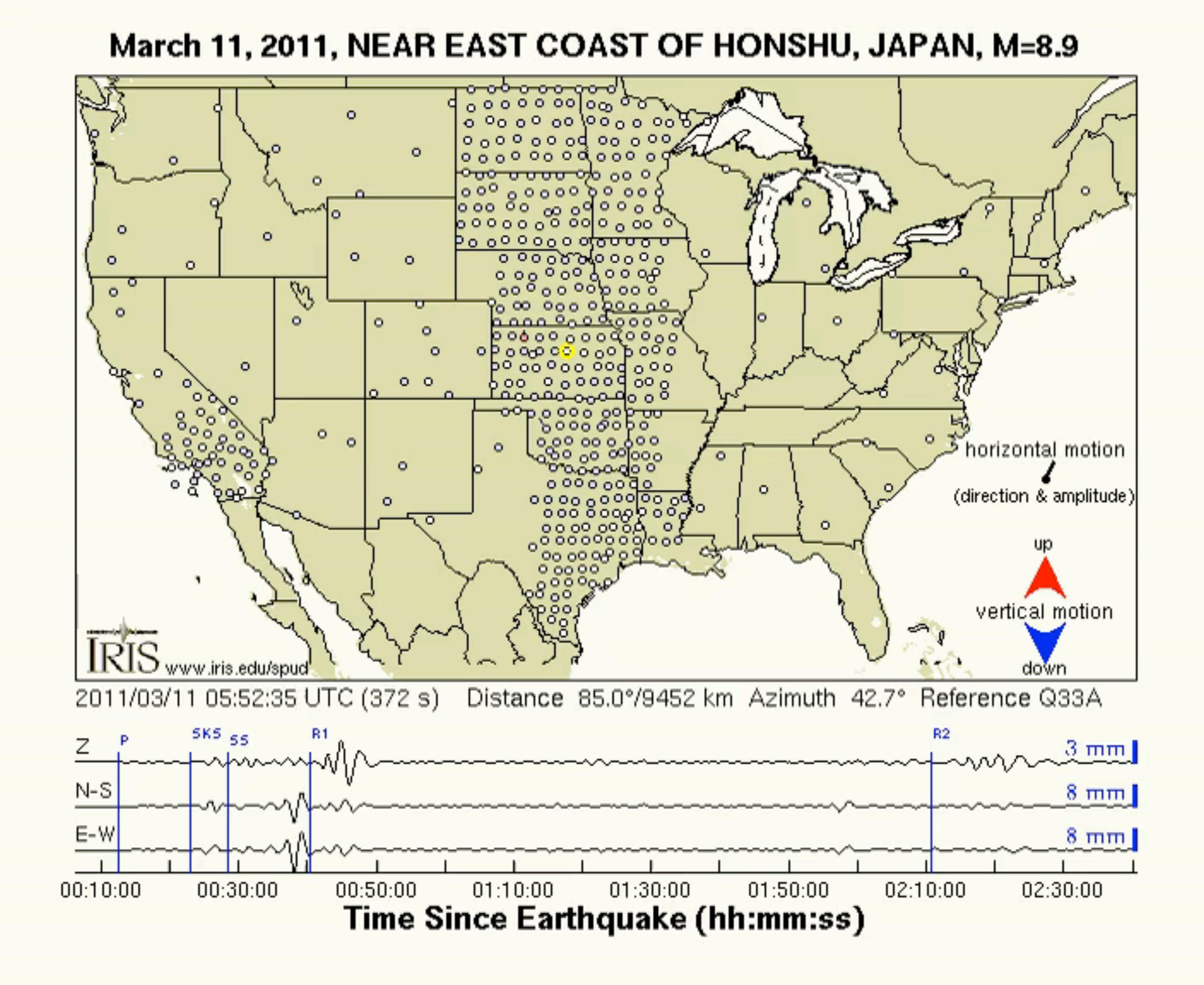




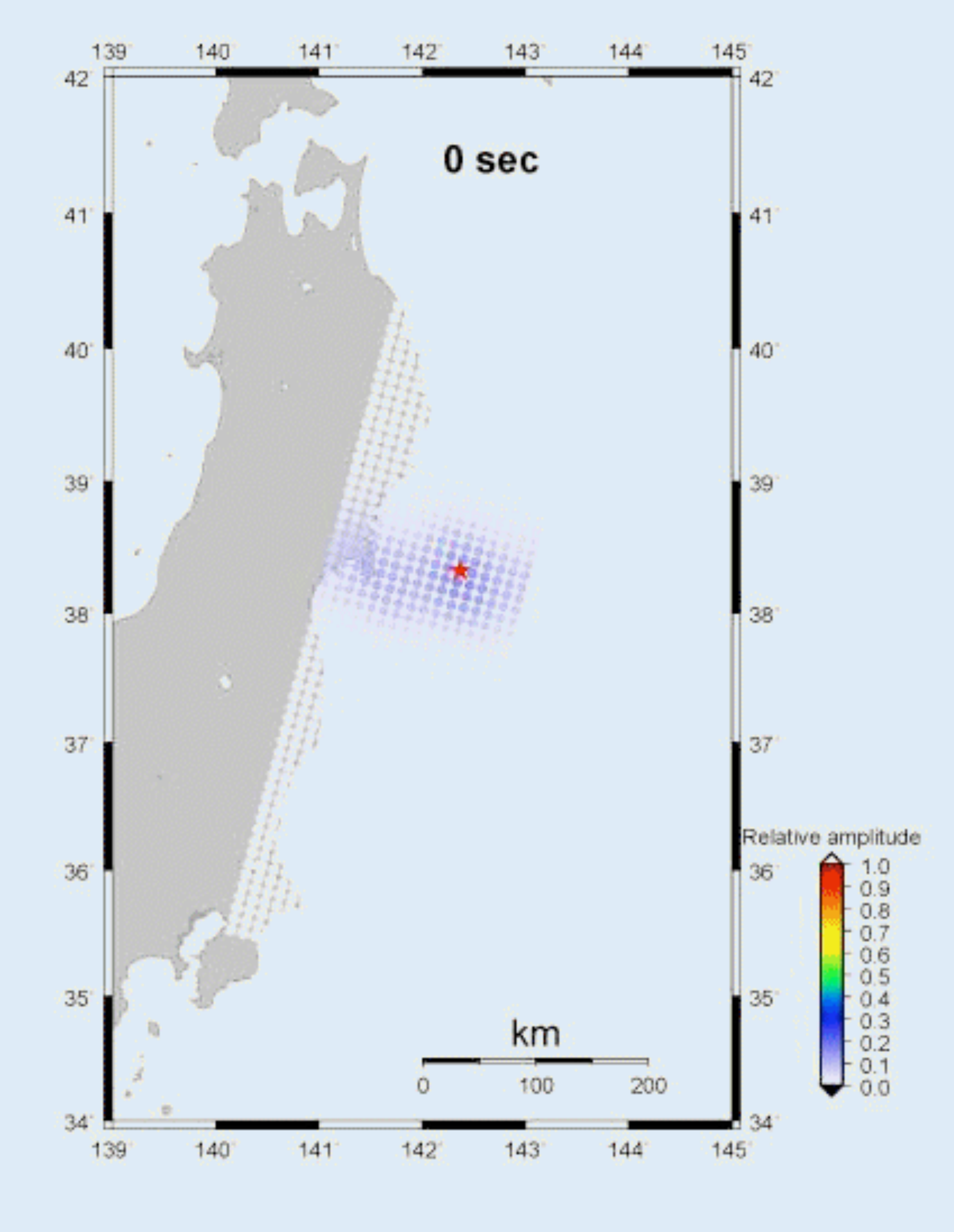




Ground motion - USA

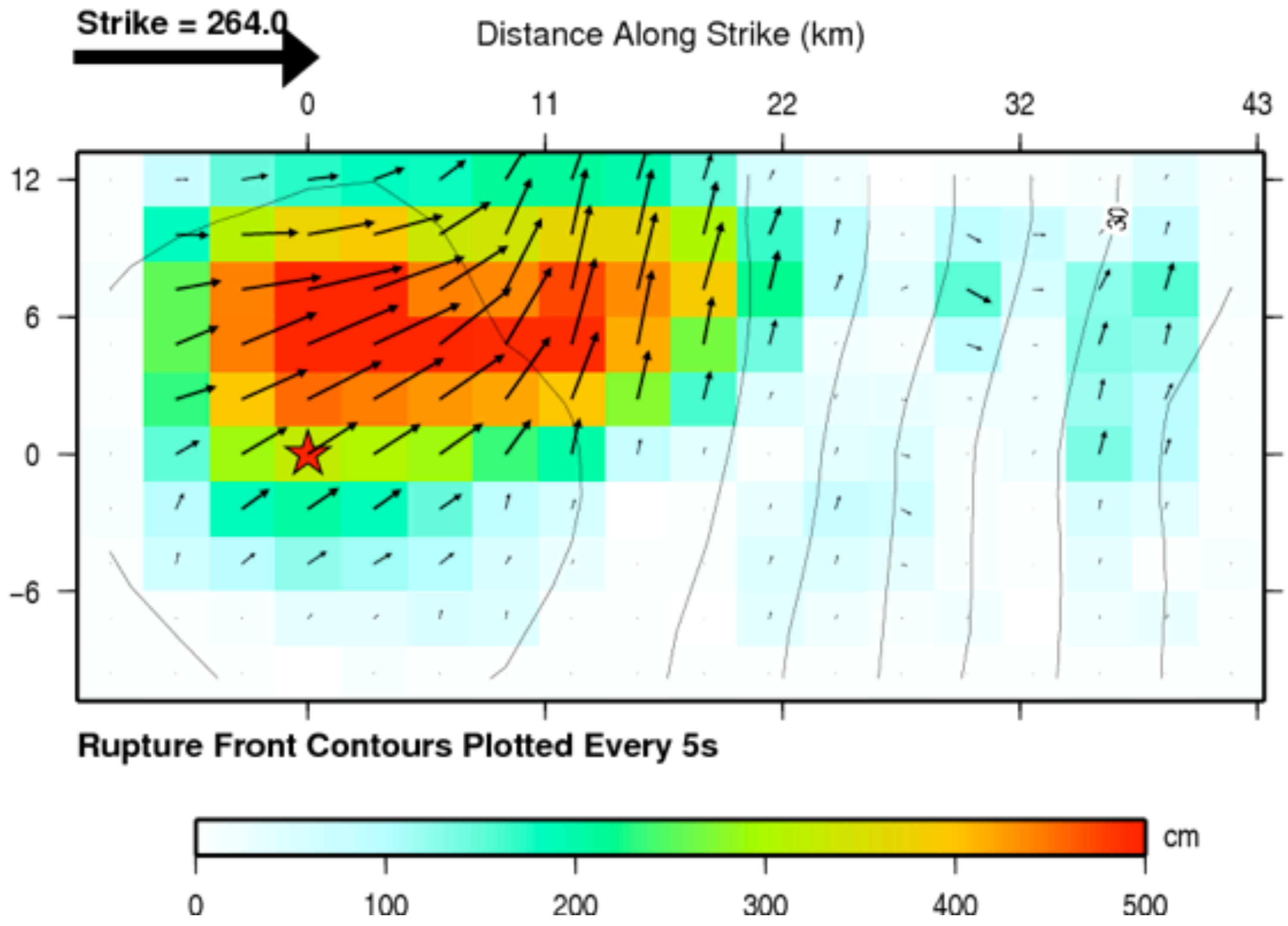


Finite fault model from backprojection



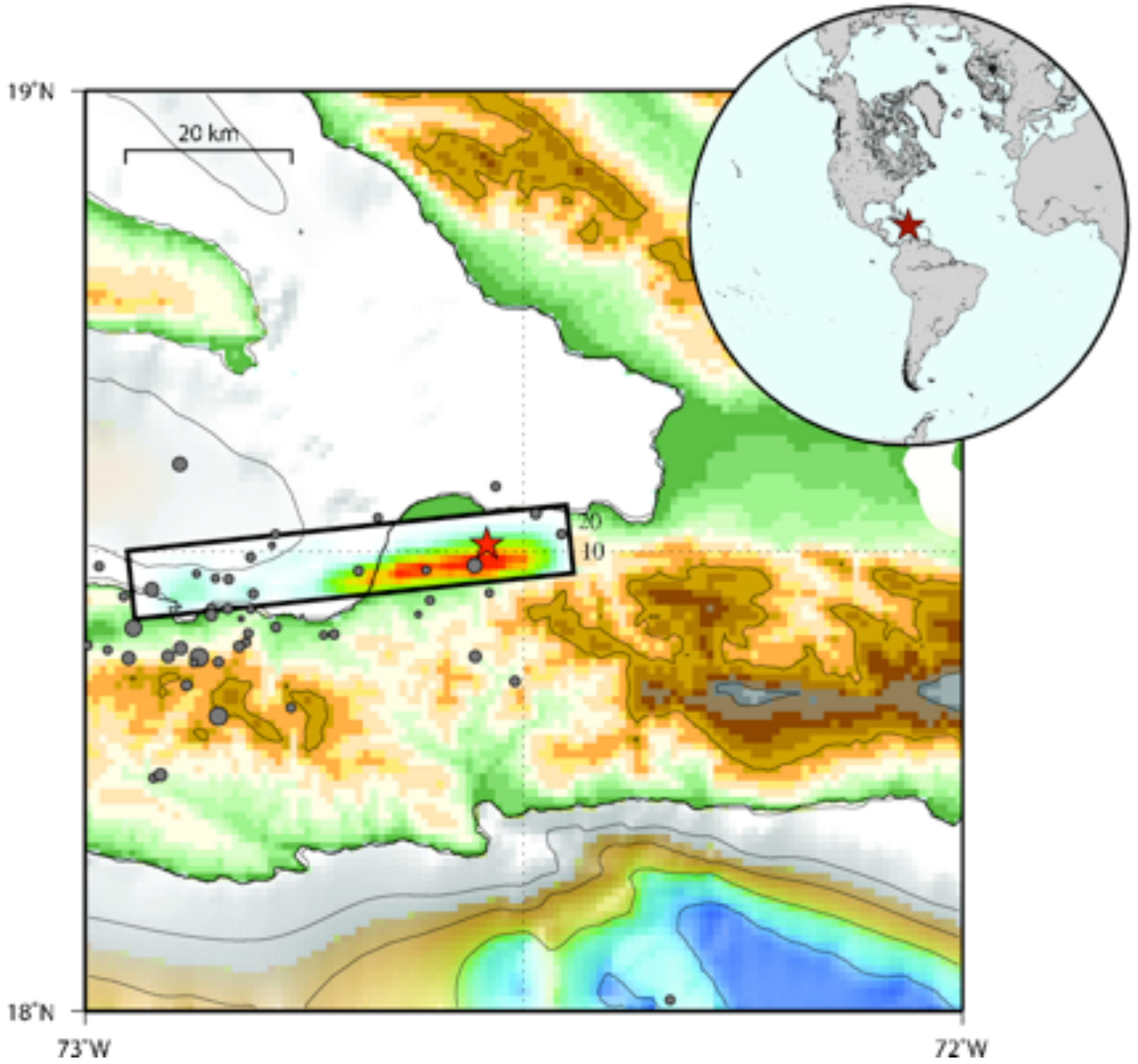
Courtesy of Dun Wang and Jim Mori

Rupture process



Cross-section of slip distribution for the Jan 12, 2010 Mw 7.0 Haiti Earthquake.

The strike direction of fault plane is indicated by the black arrow and the hypocenter location is denoted by the red star. The slip amplitude are showed in color and motion direction of the hanging wall relative to the footwall is indicated by arrows. Contours show the rupture initiation time in seconds.

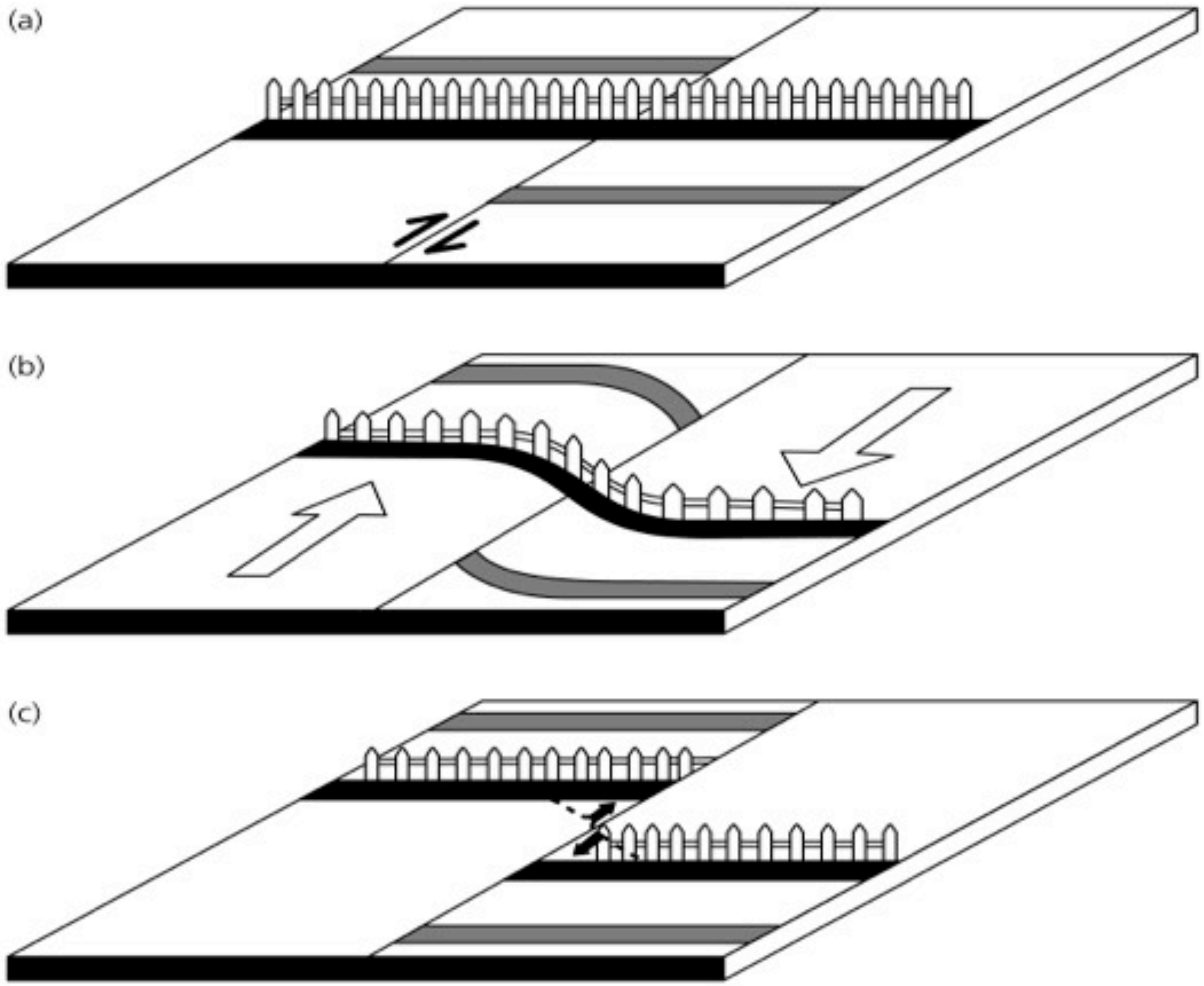


Surface projection of the slip distribution superimposed on ETOPO2.

The dark gray circles indicate the locations of ~20 hours of aftershocks

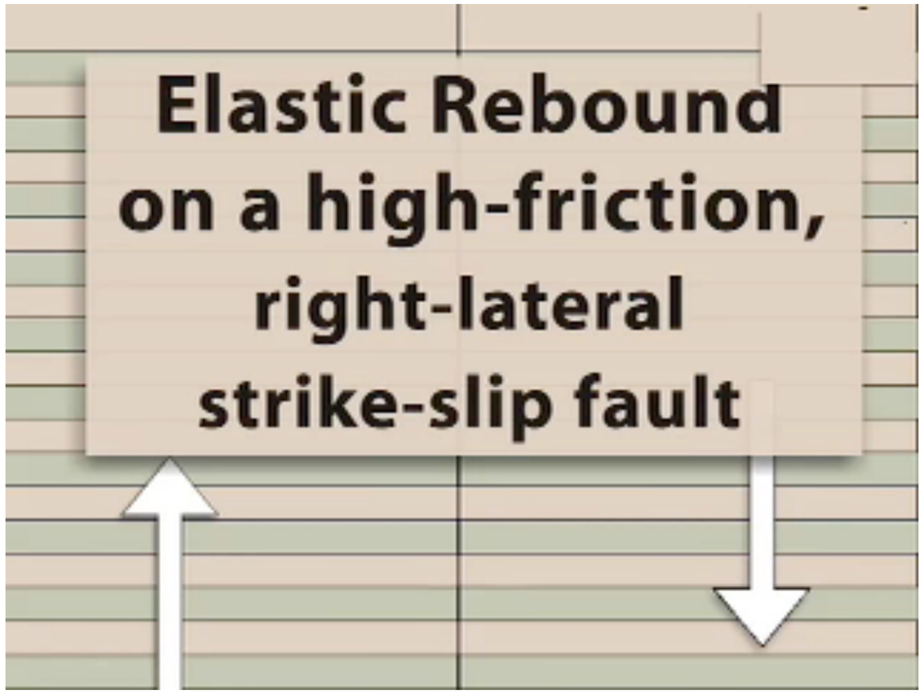
Fault slip involves 3 main stages: 1) initiation of fault sliding; 2) rupture front expansion; 3) termination of rupture process. Slipping points radiate outgoing seismic waves.
In general, rupture wavefront is not regular and slip vector, as well as slipping time, is different for the points on the fault.

Elastic rebound (Reid)



From an examination of the displacement of the ground surface which accompanied the 1906 San Francisco earthquake, Henry Fielding Reid, Professor of Geology at Johns Hopkins University, concluded that the earthquake must have involved an "elastic rebound" of previously stored elastic stress.

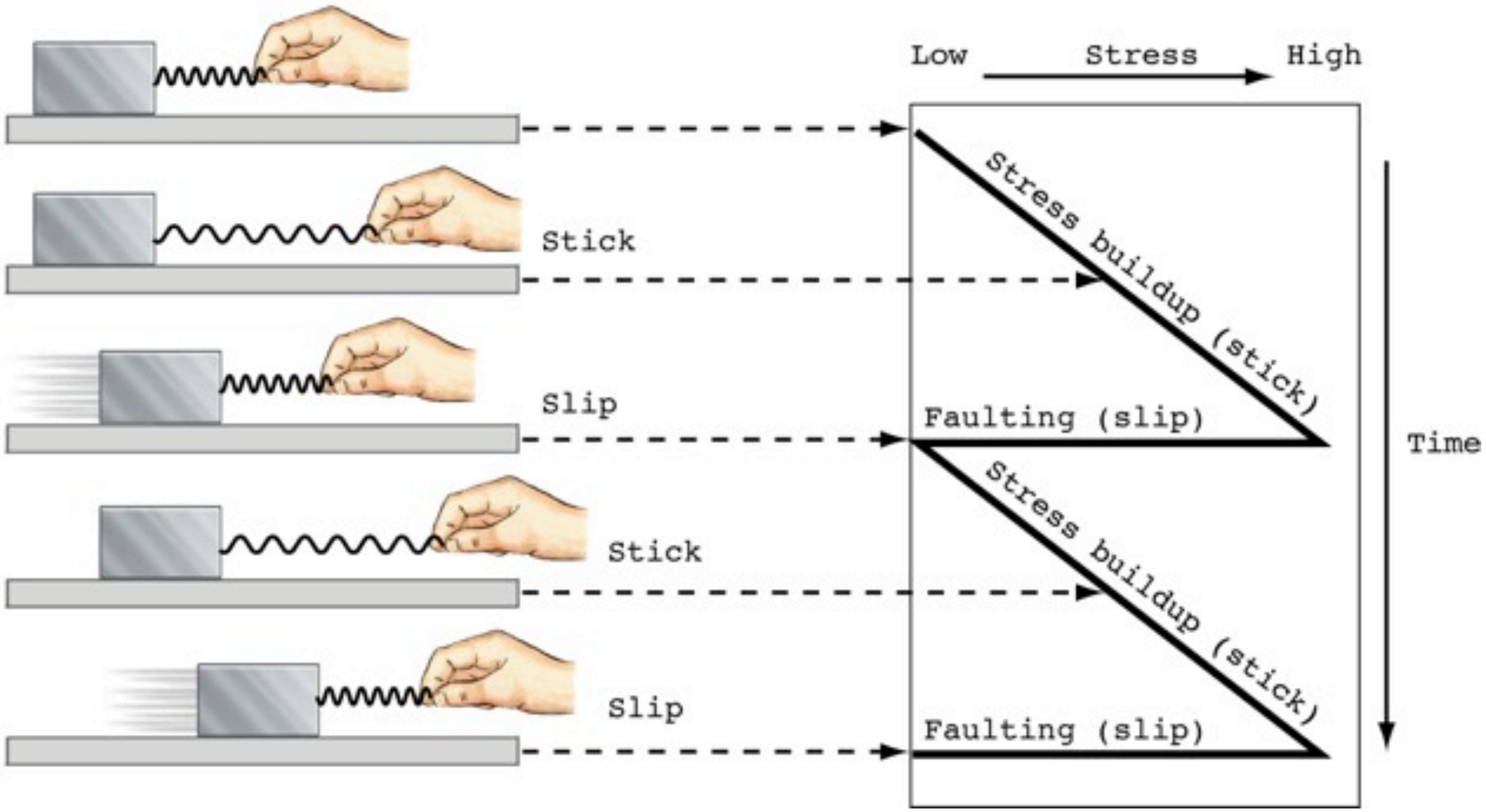
Reid, H.F., “The mechanics of the earthquake”, v. 2 of “The California earthquake of April 18, 1906”. Report of the State Earthquake Investigation Commission, Carnegie Institution of Washington Publication 87, 1910.



http://www.iris.edu/hq/programs/education_and_outreach/aotm/4

Figure 4.1-4: Displacement of crops rows during the 1979 Imperial fault earthquake.

Stick-slip

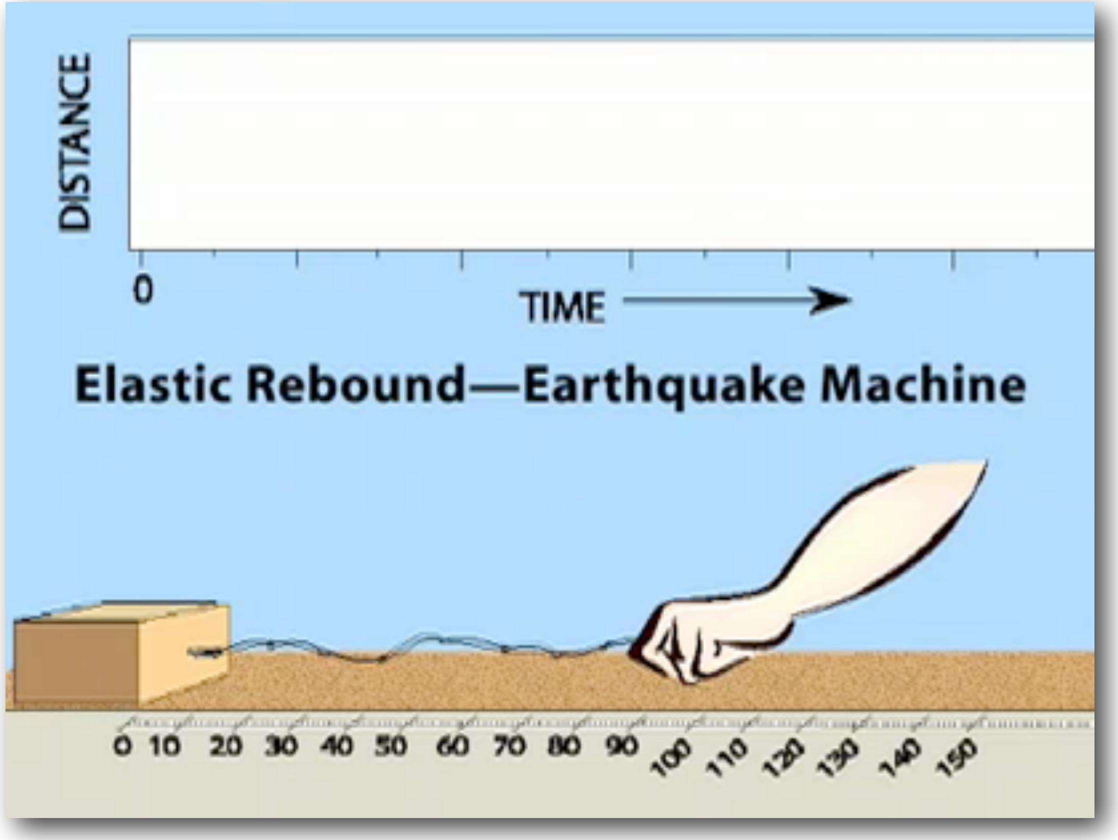


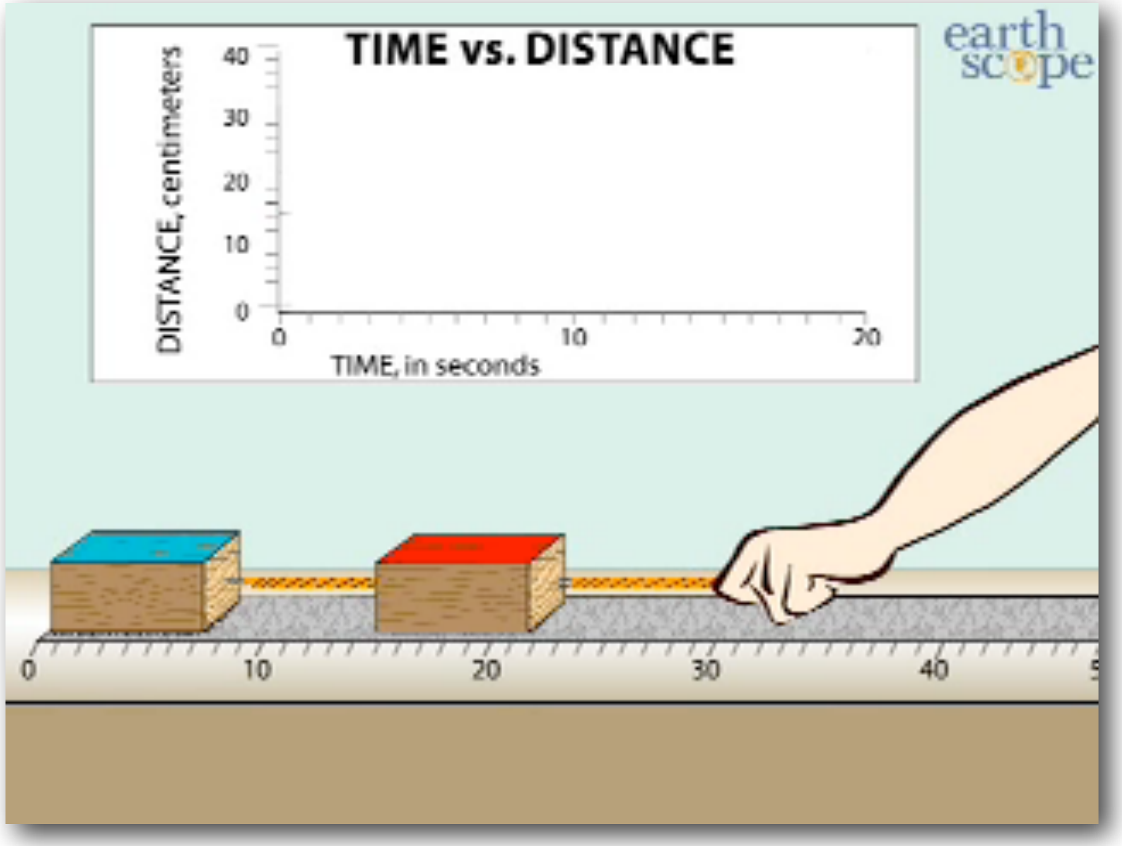
Earth, S. Marshak, W.W. Norton

Elastic strain accumulates during the interseismic period and is released during an earthquake. The elastic strain causes the earthquake –in the sense that the elastic energy stored around the fault drives earthquake rupture.
There are three basic stages in Reid's hypothesis.

1) Stress accumulation (e.g., due to plate tectonic motion)
2) Stress reaches or exceeds the (frictional) failure strength
3) Failure, seismic energy release (elastic waves), and fault rupture propagation

Stick-slip





http://www.iris.edu/hq/programs/education_and_outreach/aotm/1

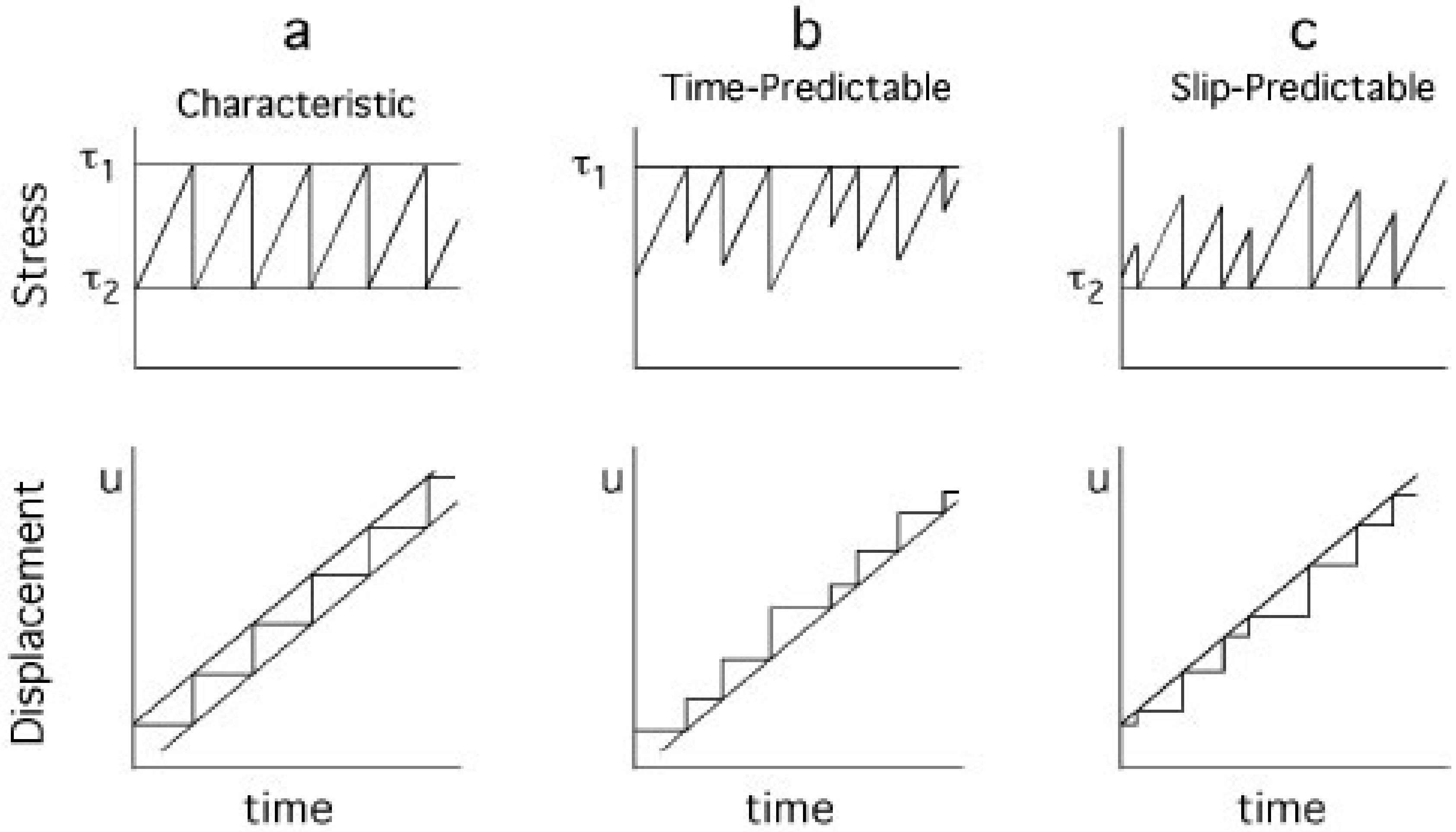
Stress cycle: prediction models

τ_1 is the shear stress at initiation of slip and reflects fault strength. τ_2 is the shear stress at which slip ceases and reflects fault friction.

(a) **Characteristic model** of stick-slip faulting. Each earthquake is identical in stress history, recurrence interval and slip.

(b) **Time-predictable model**. If slip is proportional to stress drop, and plate motions are steady, we can predict the time of the next earthquake based on the amount of slip during the previous earthquake.

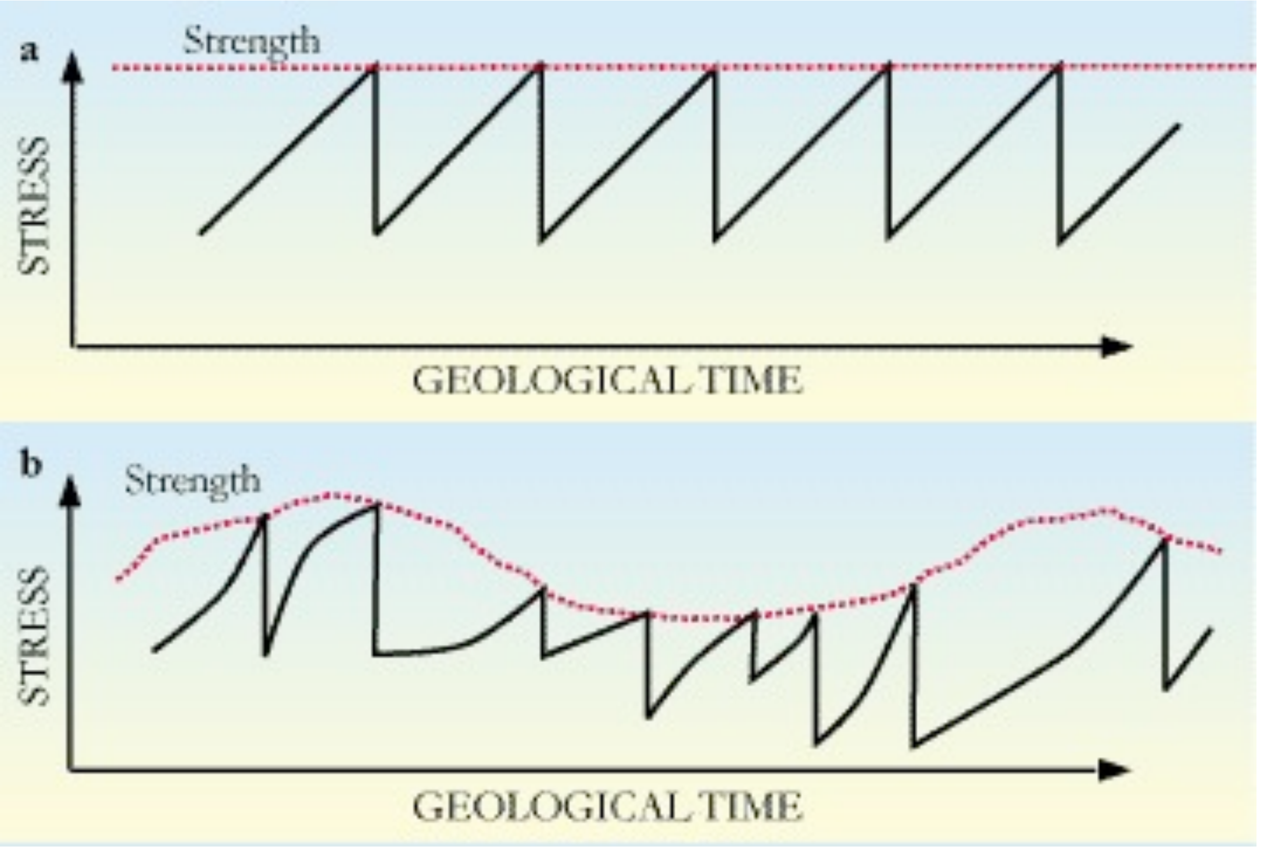
(c) **Slip-predictable model**. Knowing the time of the last earthquake and assuming steady plate motion, we can predict the size of an earthquake expected at a particular time.



Stress cycle

The stress drop causes a time interval during which the stress builds up again to critical value. This type of frictional behaviour is known as **stick-slip**, or unstable sliding (as opposed to continue slip on smooth surfaces: stable sliding).

Earthquakes are generally thought to be **recurring slip episodes on preexisting faults**: the importance is no more on the strength of the rock but on the stress-stability cycle.

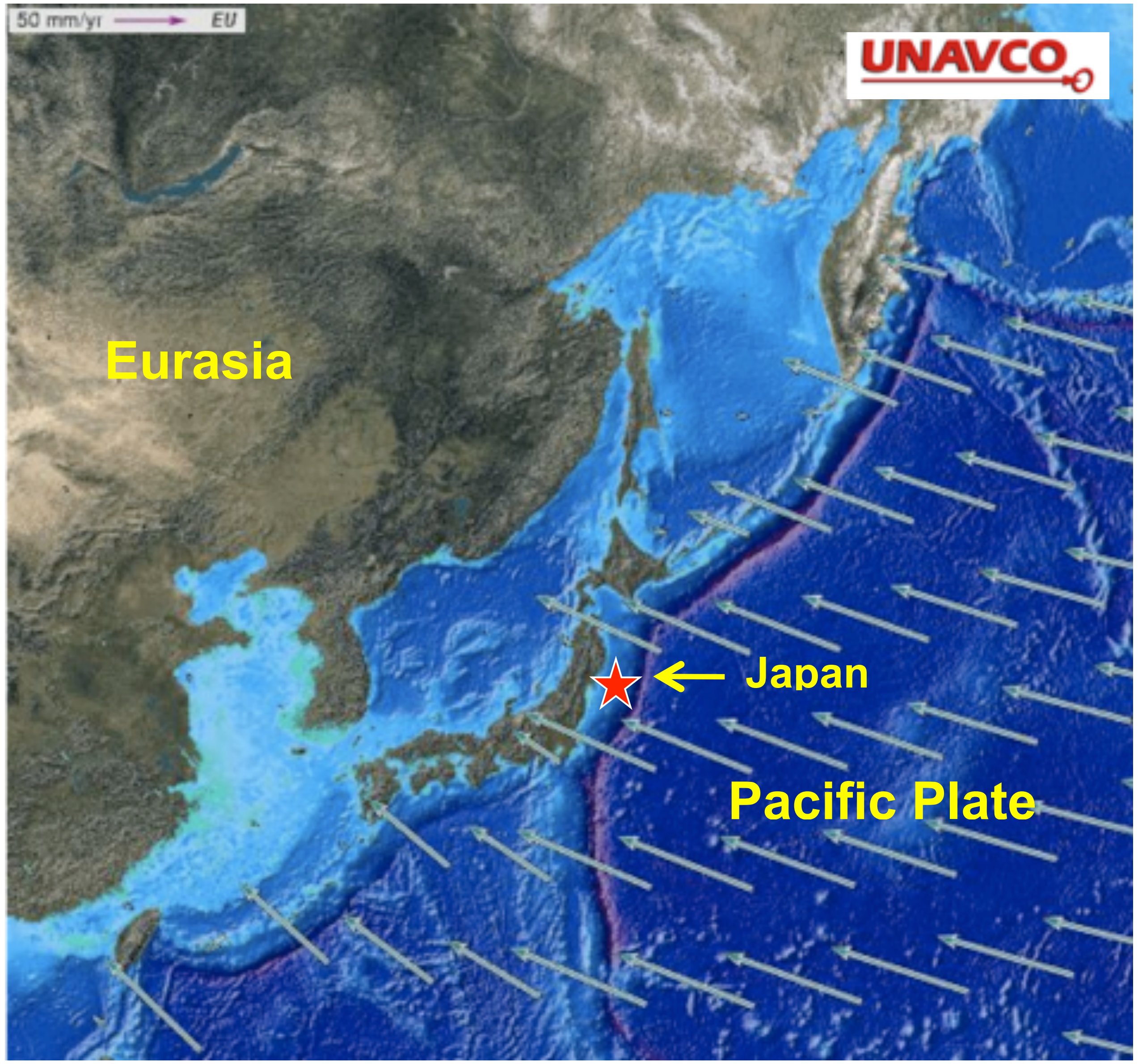


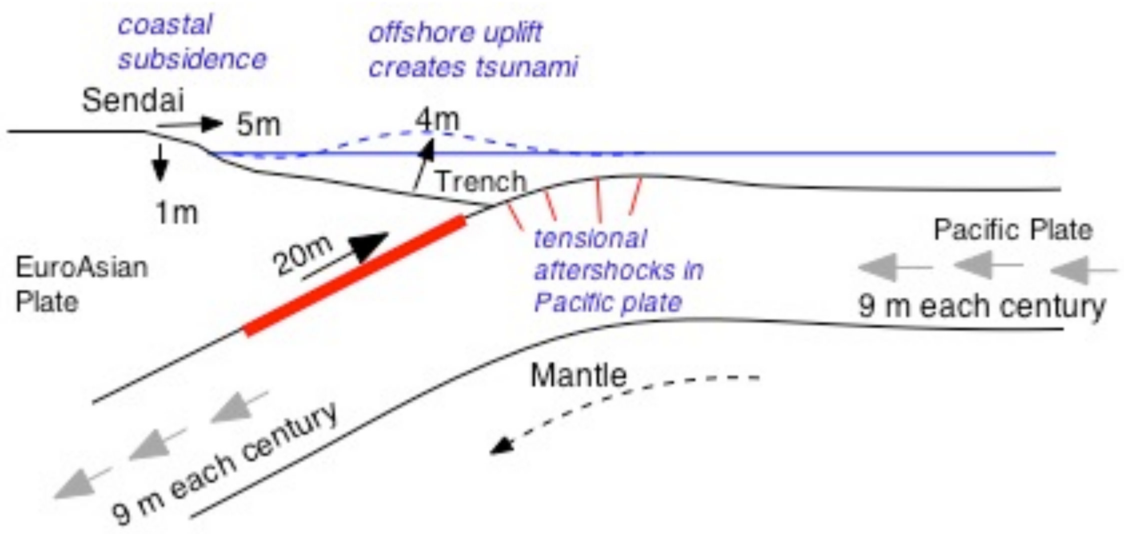
Tohoku-oki event: tectonic setting

This earthquake was the result of thrust faulting along or near the convergent plate boundary where the Pacific Plate subducts beneath Japan.

This map also shows the rate and direction of motion of the Pacific Plate with respect to the Eurasian Plate near the Japan Trench. The rate of convergence at this plate boundary is about 100 mm/yr (9 cm/year).

This is a fairly high convergence rate and this subduction zone is very seismically active.





Historical seismicity and aftershocks

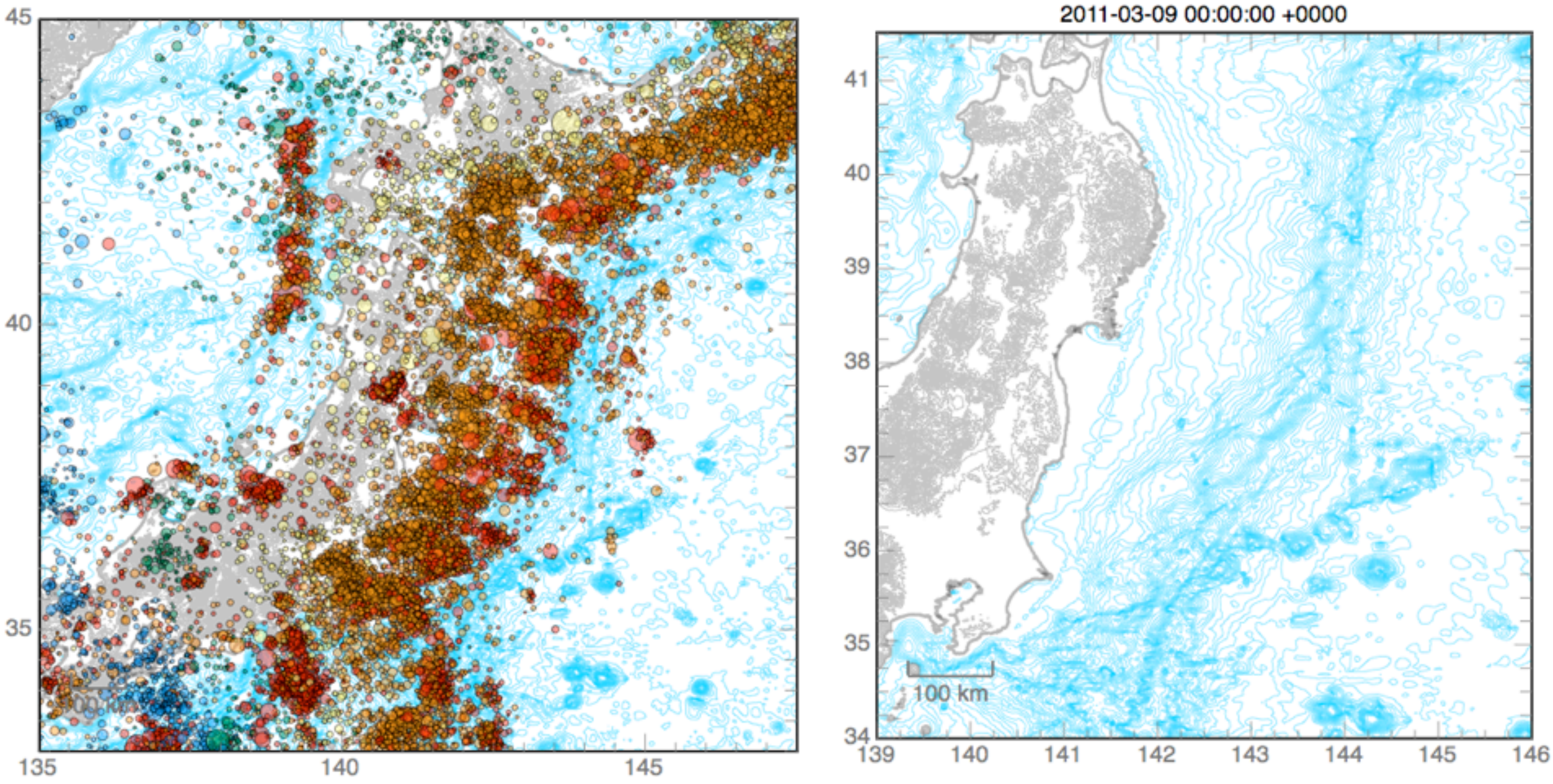
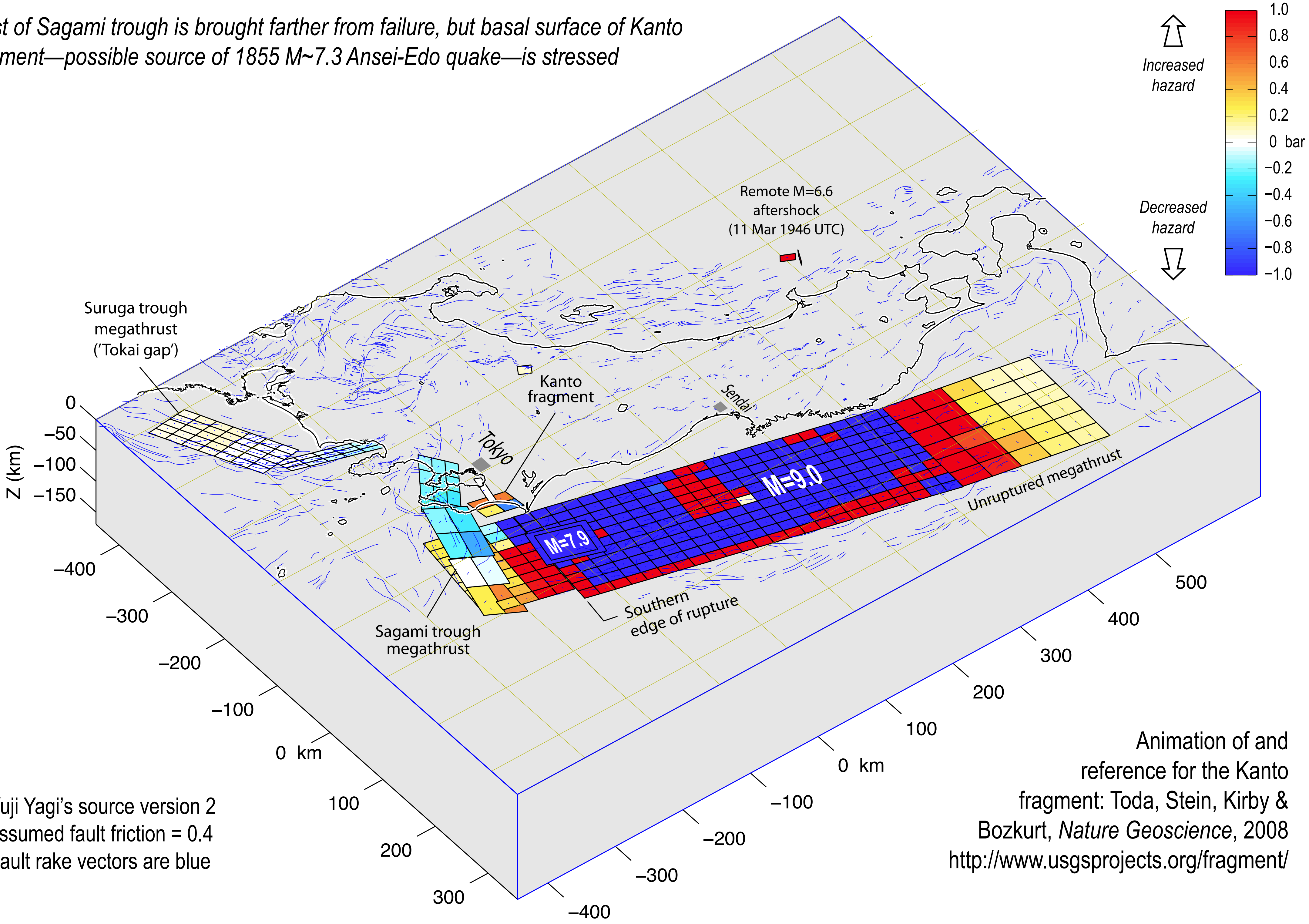


Image courtesy of Charles Ammon

Dynamic rupture and stress transfer

Coulomb stress imparted by the M=9.0 Off-Tohoku rupture and its M=7.9 aftershock to Japan Trench, Sagami Trough and Kanto Fragment

Most of Sagami trough is brought farther from failure, but basal surface of Kanto fragment—possible source of 1855 M~7.3 Ansei-Edo quake—is stressed

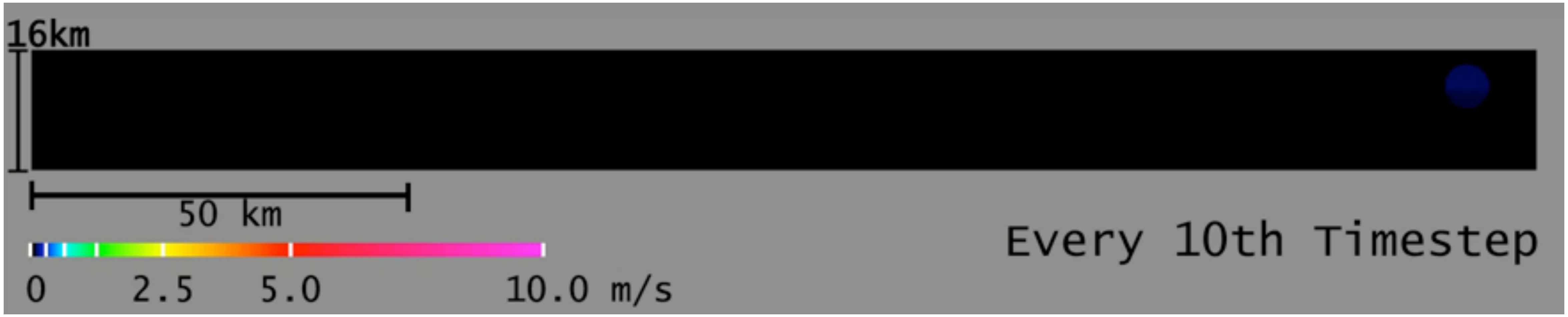


Yuji Yagi's source version 2
assumed fault friction = 0.4
Fault rake vectors are blue

Animation of and reference for the Kanto fragment: Toda, Stein, Kirby & Bozkurt, *Nature Geoscience*, 2008
http://www.usgsprojects.org/fragment/

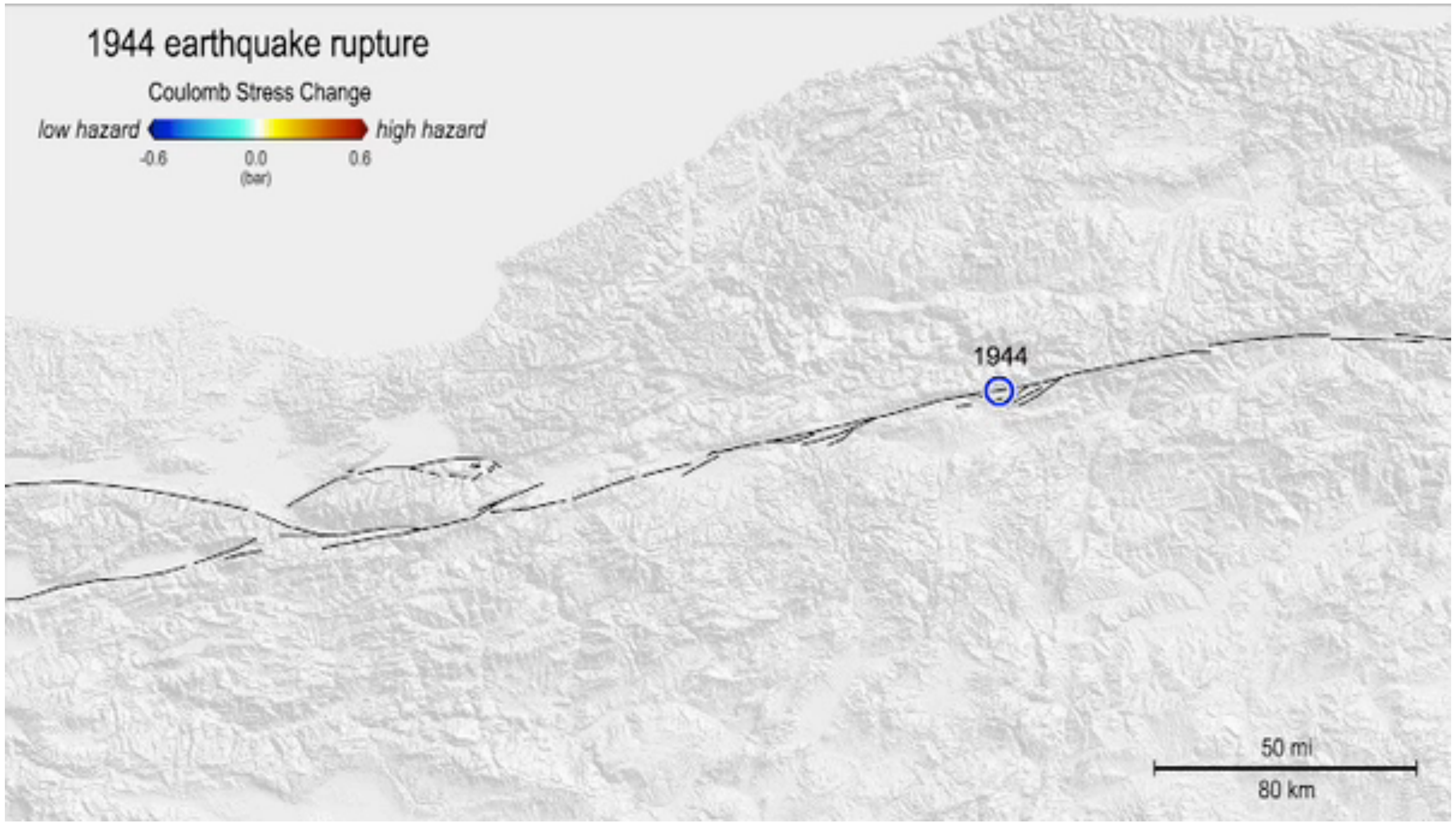
Ross Stein & Volkan Sevilgen (USGS) and Shinji Toda (DPRI, Kyoto Univ.) 18 Mar 2011 4:15 PM PDT

Dynamic rupture and stress transfer



rupture velocities of a dynamic rupture model of a magnitude 7.7 on the southernmost San Andreas fault www.scec.org

When a fault fails during an earthquake, it modifies the stress field in its surroundings. The modification of the stress pattern can give a rough idea of where the next shocks are more likely occur.



Coulomb stresses transmitted by seismic wave propagation for the M=7.2 1944 earthquake on the North Anatolian fault.

Courtesy of Kim B. Olsen

http://visservices.sdsc.edu/projects/scec/terashake/compare/

How does brittle deformation take place?

solid composed of atoms or ions bonded to one another through chemical bonds which can be visualized as tiny springs

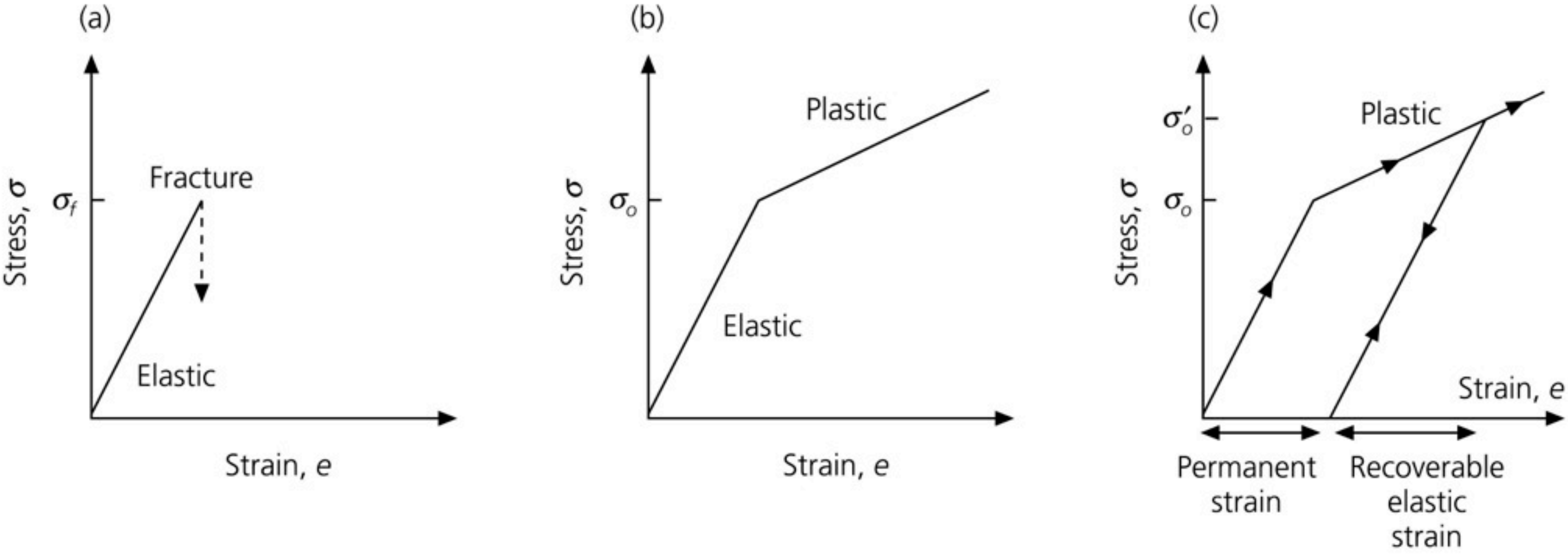
- each chemical bond has an equilibrium length
- any two chemical bonds connected to same atom have an equilibrium angle between them

during elastic strain.....bonds holding atoms together in solid, stretch, shorten, and/or bend, but they do not break...
once stress is removed, the bonds return to equilibrium...
elastic strain is recoverable

rock cannot develop large elastic strains (only a few percent)
...must deform in a ductile way (does not break)
...must deform in a brittle way (does break)

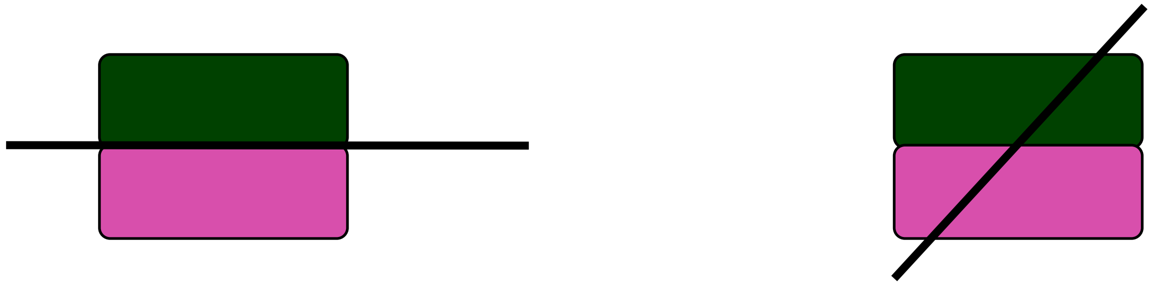
Different rheology

Figure 5.7-1: Elastic and plastic rheologies.



during brittle deformation… stresses become large enough to bend, then break atomic bonds….
new fracture forms or old surface slips

fractures can be between grains or across grains



what exactly happens when something breaks?

…discussing solids…. (liquids and gases don't break)

…breaks bonds at atomic scale…

Type of experiments...

axial compression:

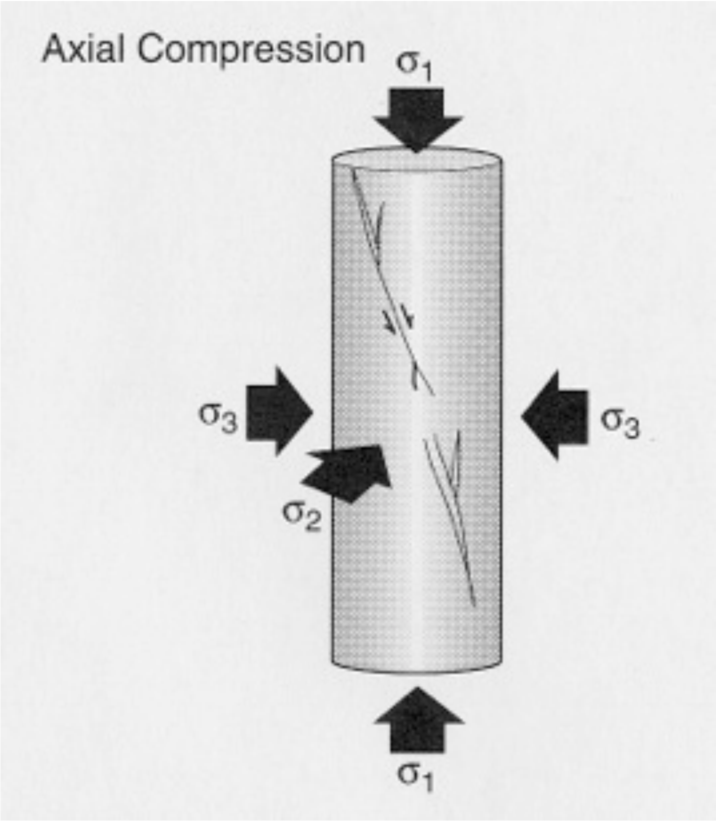
vertical axial compressive stress > confining pressure

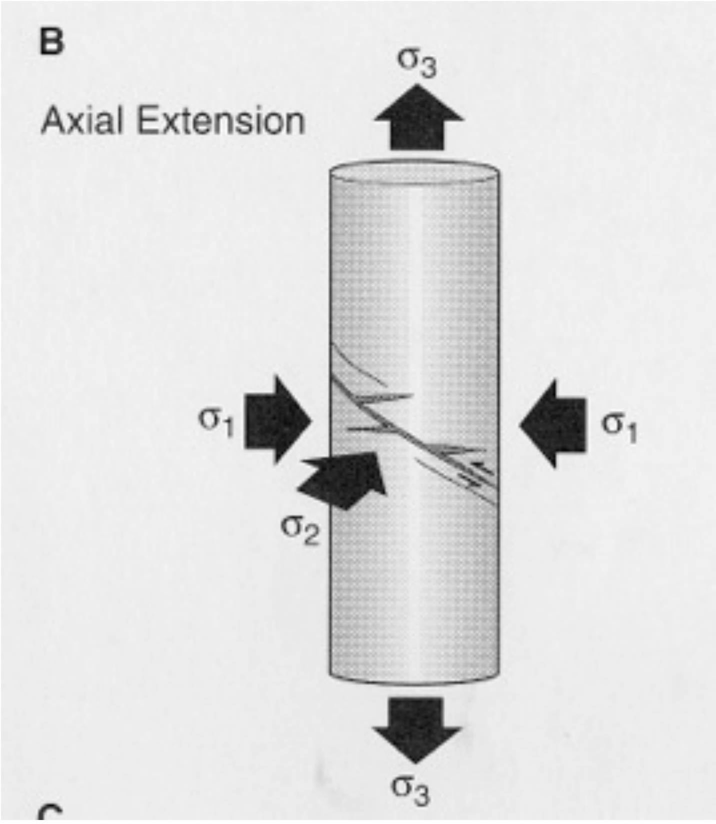
axial extension:

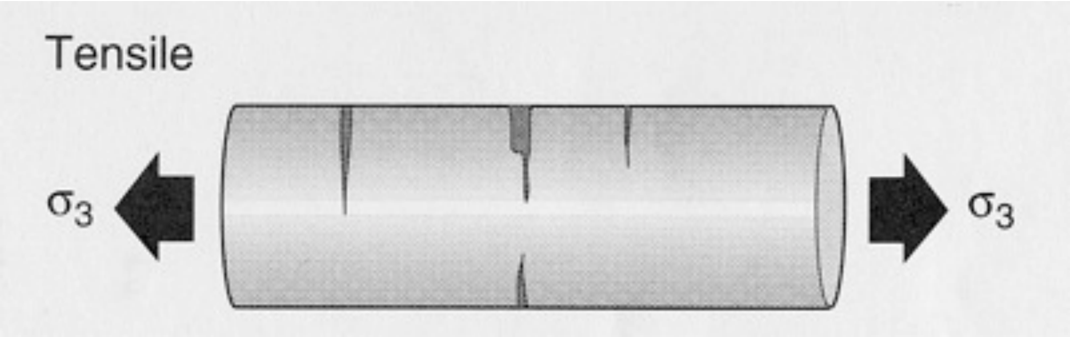
confining pressure > vertical axial compressive stress

tensile strength:

rocks pulled apart







from: Davis and Reynolds, 1996

called triaxial deformation experiments... this is misleading...

most do not permit three principal stresses to vary independently

Brittle deformation processes

1) **tensile cracking**:

opening and propagation of cracks into unfractured material

2) **shear rupture**:

initiation of macroscopic shear fracture

3) **frictional sliding**:

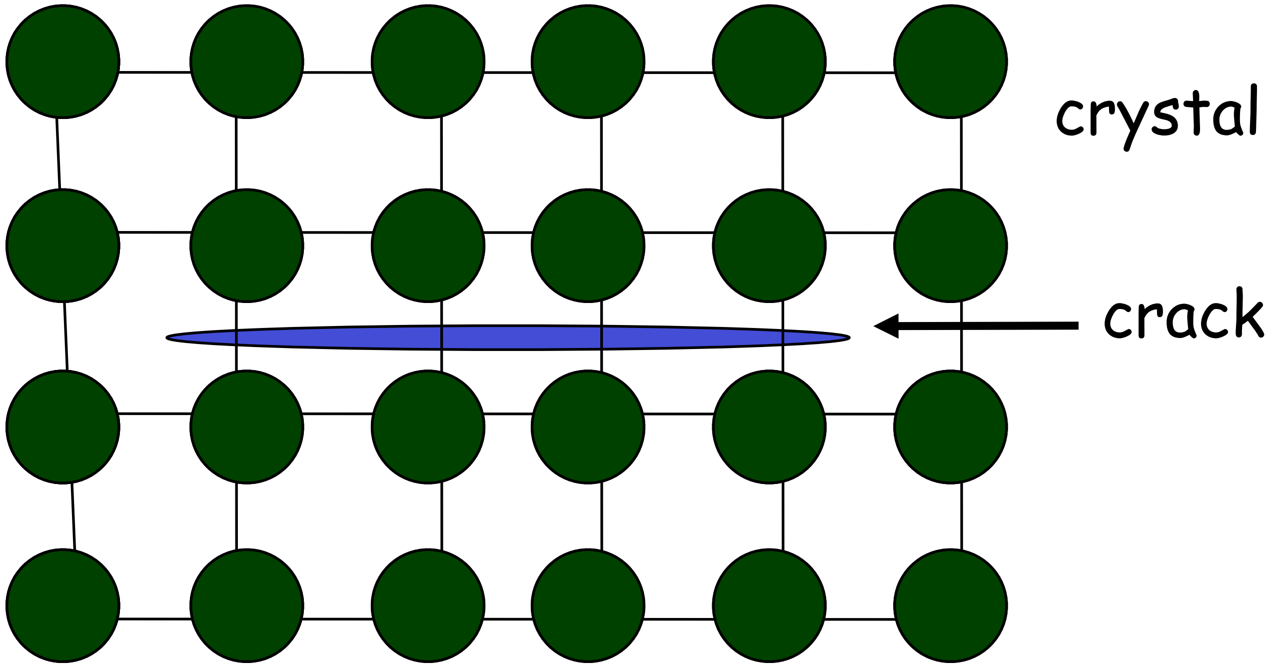
sliding on preexisting fracture

4) **cataclastic flow**:

macroscopic ductile flow from grain-scale fracturing and frictional sliding

1) Tensile cracking

cracks on atomic scale:


crystal lattice (atoms and bonds)

crack

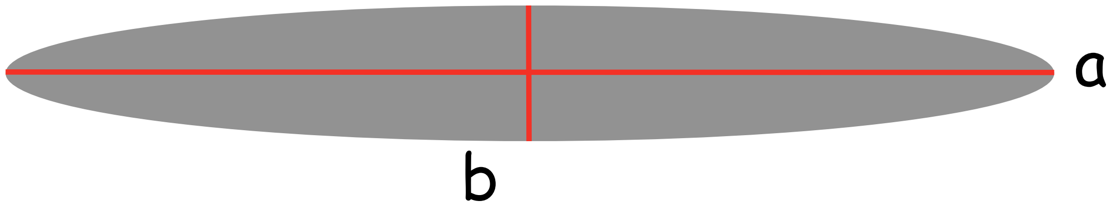
one model is for crack surface to break at once…
tensile stress necessary is equal to strength of each chemical bond multiplied by number of bonds

this theoretical strength is ~500 to 5000 MPa
….very large number!…
measurement of rock strength in Earth's crust suggests
tensile cracking occurs at about 10 MPa or less

this is known as the strength paradox

engineers realized far-field stress
(stress applied at a distance from area of interest)
is concentrated at sides of flaws (holes) in an
elastic (recoverable) medium

concentration along an ellipse-shaped flaw is
(2a + 1)/b with a as long axis and b as short axis of ellipse



stress concentration at ends of elliptical hole depend on axial ratio:
axial ratio of 8:1--concentration factor of 17
axial ratio of 32:1--concentration factor of 65

Griffith crack theory

in 1920s A.W. Griffith applied this idea to fracture formation

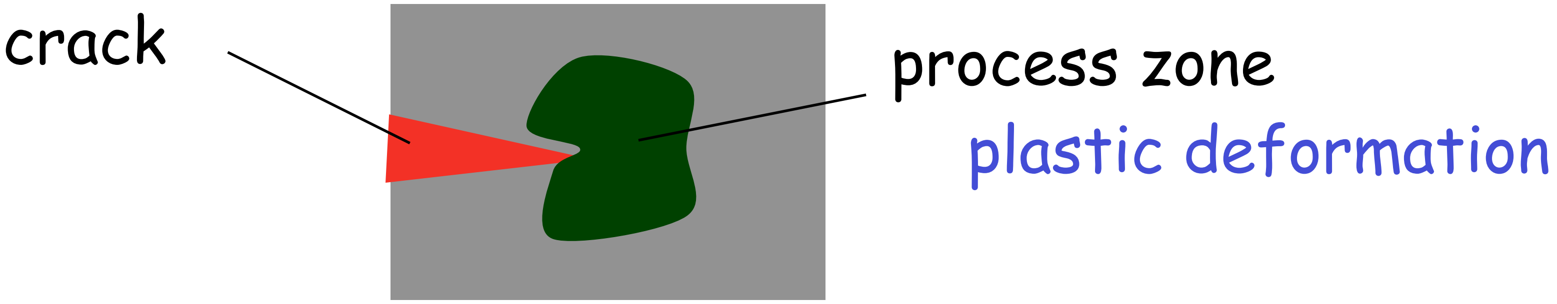
- **all materials contain preexisting microcracks where stress is concentrated**
- **microcracks propagate and grow even under low far-field stress**

crack with largest axial ratio will propagate first

rocks in Earth's crust are weak because they contain Griffith cracks

in 1930s a new approach: linear elastic fracture mechanics

…all cracks have nearly infinite axial ratio (cracks are sharp)
…do not propagate under very small stresses because tips are blunted by a crack-tip process zone



predicts that a longer crack will propagate before a shorter one

Griffith crack theory and linear elastic fracture mechanics

imply cracks do not form instantaneously….

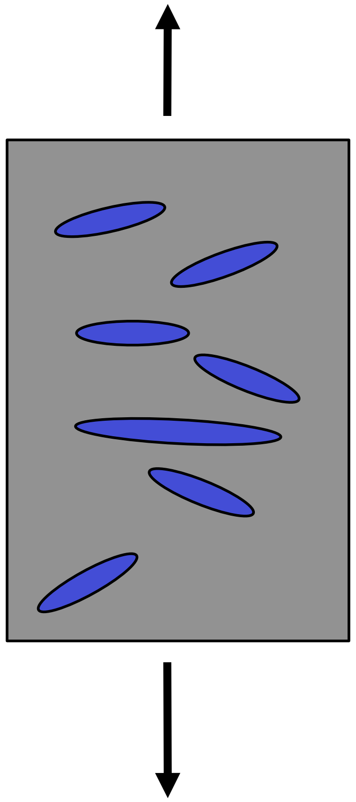
begin at small flaw and grow outward

not all bonds break at once → theoretical strength is not reality

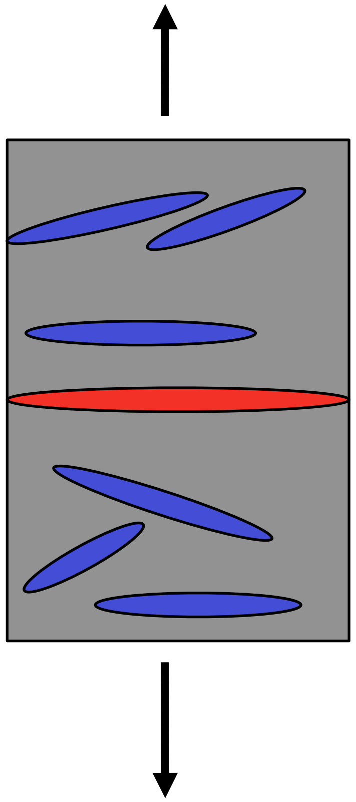
what happens during tensile cracking?

…look at laboratory experiments

rock cylinders stretched along axis



opening of
microcracks



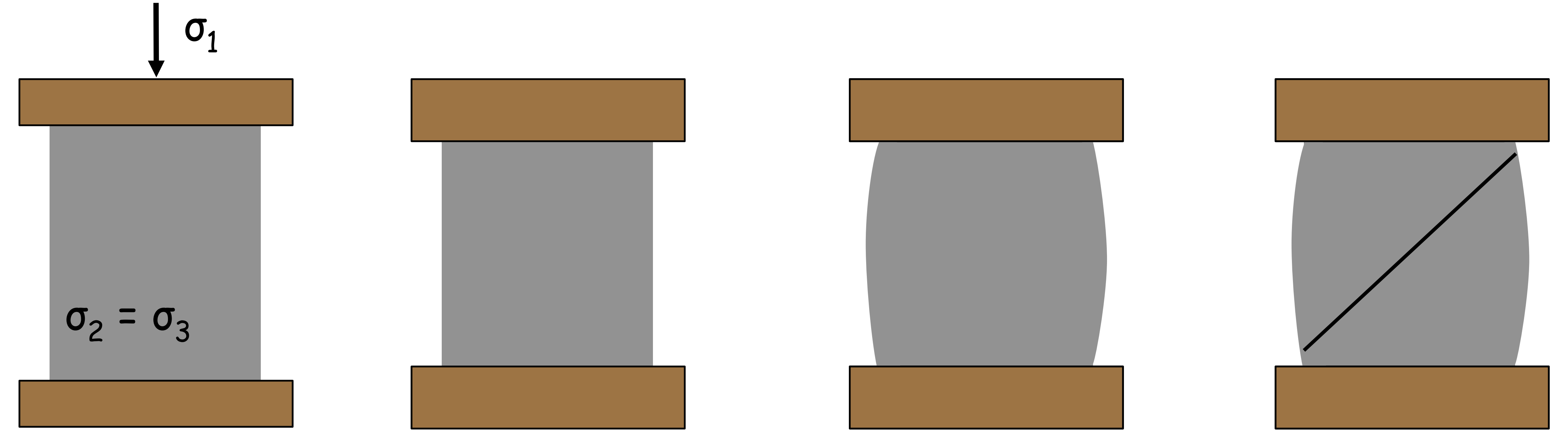
largest crack forms
throughgoing crack
(when crack reaches
edges of sample, the
sample separates into
two pieces)

2) Shear rupture (fracture)

surface across which rock loses continuity when shear stresses parallel to surface are sufficiently large

in rock cylinder experiments, shear fractures form at acute angle to far-field σ_1 $(\sigma_1 > \sigma_2 = \sigma_3)$

normal stress component across surface generates frictional resistance; if shear stress component exceeds resistance → evolve into fault



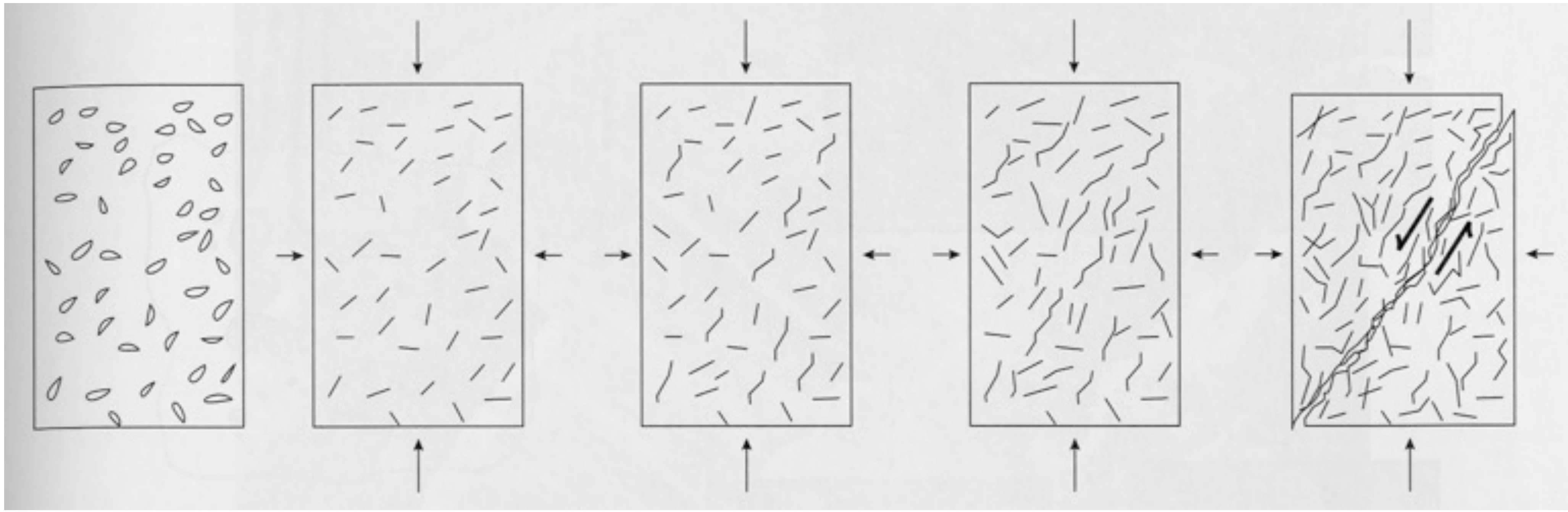
laboratory triaxial-loading

volume decreases

volume increases (dilatancy-cracking)

cracks coalesce to form fault (failure)

what happened in the rock cylinder during experiment?



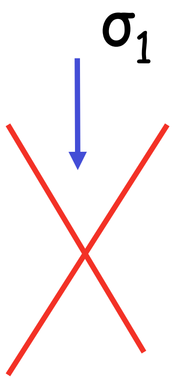
cracks form

cracks coalesce

from: van der Pluijm and Marshak, 1997

failure strength for shear fracture: not a definition of stress state when single crack propagates, but when
many cracks coalesce to form throughgoing rupture

two shear ruptures can form (conjugates):
each at 30° to axial stress; angle between two is 60°
acute bisectrix of fractures parallels far-field σ_1



in reality, only one orientation will continue as it offsets other

3) Frictional sliding

friction: resistance to sliding on surface

frictional sliding: movement on surface occurs when
shear stress parallel to surface > frictional resistance to sliding

frictional resistance to sliding proportional to
normal stress component across surface

why? …fault surfaces have bumps on them (asperities)
that act to hold rock surfaces in place
…increase in normal stress pushes asperities into
opposing wall more deeply

4) cataclasis and cataclastic flow

cataclasis: microfracturing, frictional sliding of grains, and
rotation and transport of grains
-- similar to grinding corn between two mill stones:
grains roll, rotate, break, and grind into cornmeal

Stress

If the coordinate axes $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2)$ are oriented in the principal stress directions, the stress tensor is diagonal,

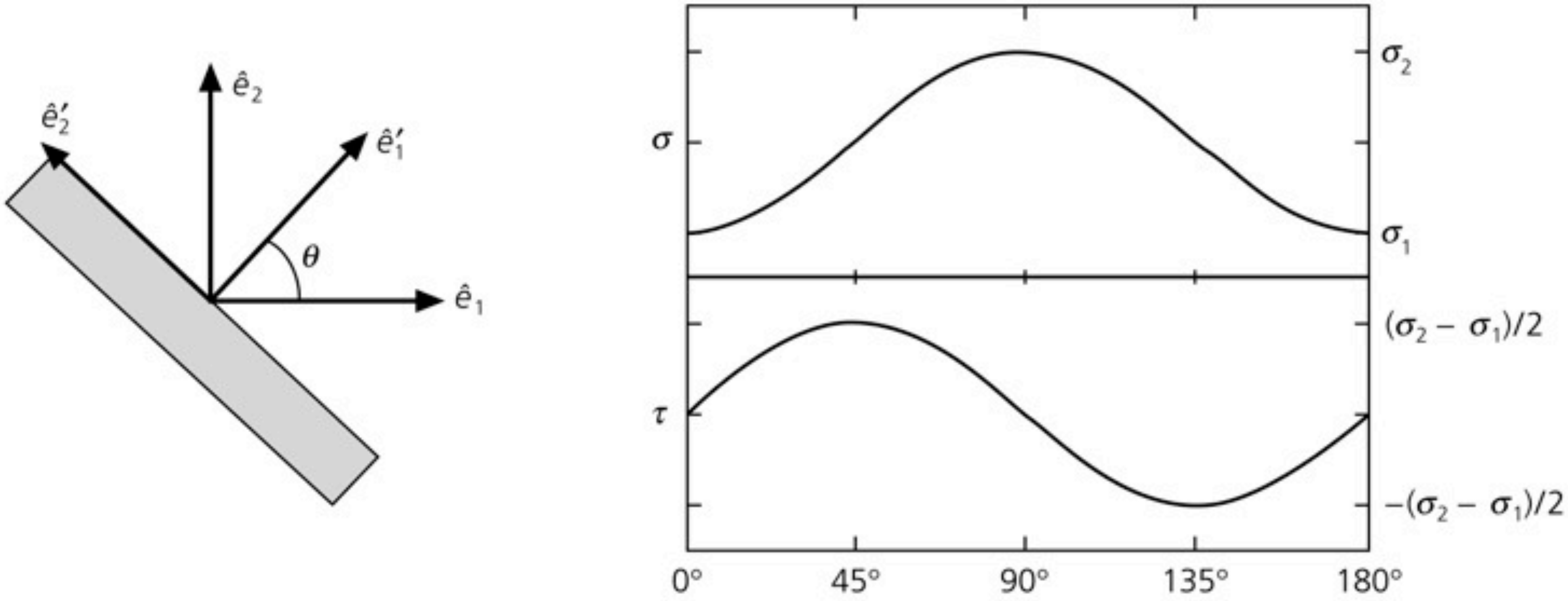
$$\sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

Now rotate the coordinate system by an angle θ: $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$\sigma'_{ij} = A\sigma A^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \sigma_1\cos^2\theta + \sigma_2\sin^2\theta & (\sigma_2 - \sigma_1)\sin\theta\cos\theta \\ (\sigma_2 - \sigma_1)\sin\theta\cos\theta & \sigma_1\sin^2\theta + \sigma_2\cos^2\theta \end{pmatrix}$$

Stress - 2

Figure 5.7-4: Normal and shear stresses as a function of geometry.



Normal stress:

$$\sigma = \sigma_{11}' = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2}\cos 2\theta$$

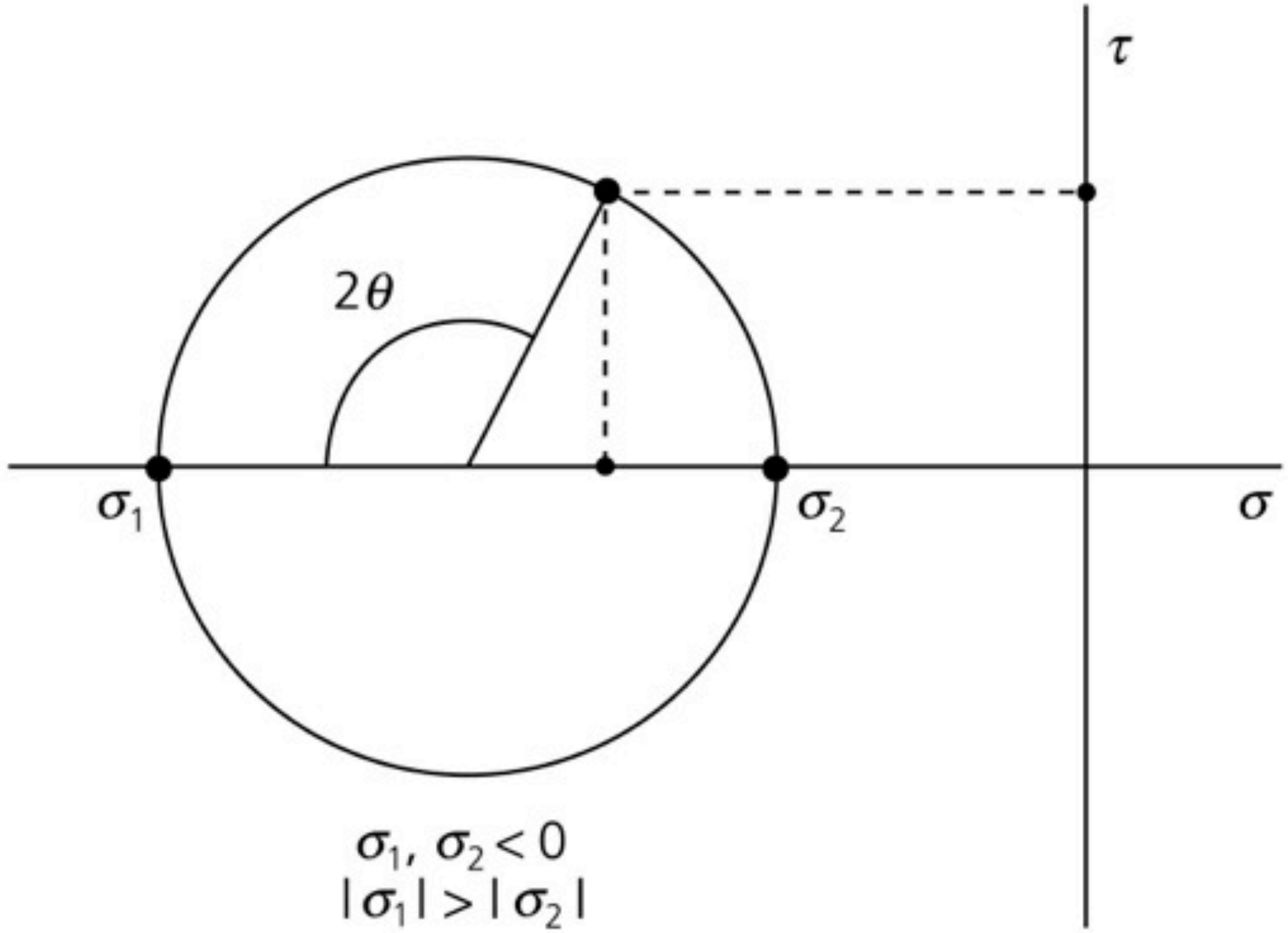
Shear stress:

$$\tau = \sigma_{12}' = (\sigma_2 - \sigma_1)\sin\theta\cos\theta = \frac{(\sigma_2 - \sigma_1)}{2}\sin 2\theta.$$

Mohr's circle shows the values of σ and τ as functions of θ (the angle between the normal to a plane and the principal stress direction, σ_1.

Mohr's circle

Figure 5.7-5: Definition of Mohr's circle.



Normal stress:

$$\sigma = \sigma'_{11} = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2}\cos 2\theta$$

Shear stress:

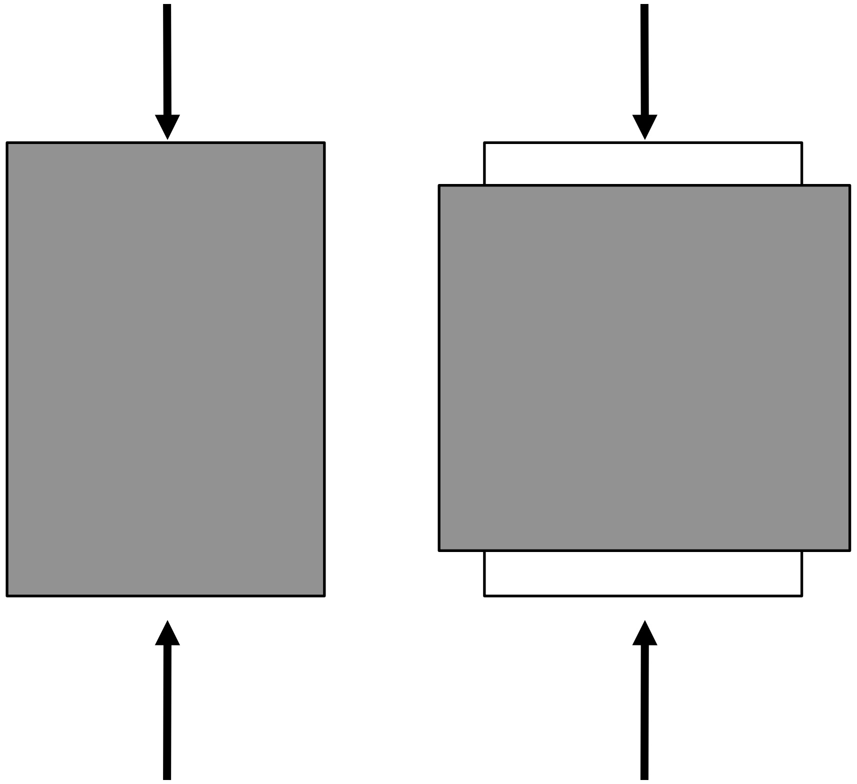
$$\tau = \sigma'_{12} = (\sigma_2 - \sigma_1)\sin\theta\cos\theta = \frac{(\sigma_2 - \sigma_1)}{2}\sin 2\theta.$$

Mohr's circle shows the values of σ and τ as functions of θ (the angle between the normal to a plane and the principal stress direction, σ_1.

2) Shear fracture

let us return to rock cylinder laboratory experiments…

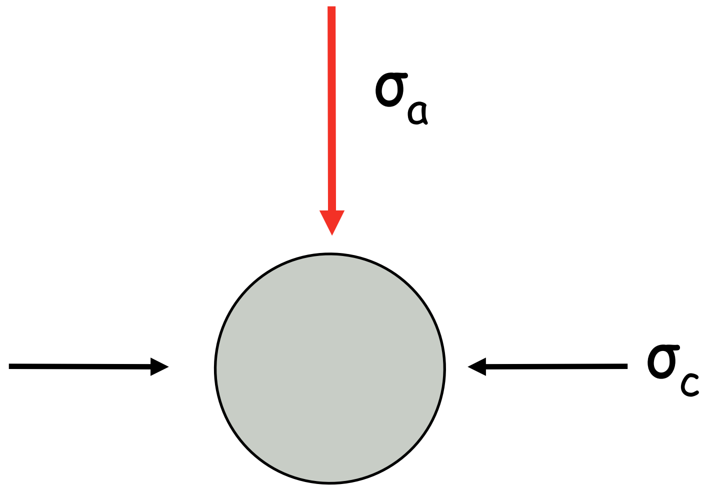
- piece of rock cut into cylinder with length 2-4 times diameter;
- sample placed between two steel pistons which are forced together;
- applied stress changes length, diameter, volume of sample, which are measured by strain gauges attached to sample



at first, when stress is removed, sample returns to original shape: recoverable

characteristic of elastic deformation (rubber band) but, if enough stress is applied, sample fractures (breaks)

…conduct triaxial loading experiments to determine applied stress at which sample breaks



σ_a= axial stress, σ_1

σ_c= confining stress, σ_3

first experiment… set confining pressure low and increase axial load (stress) until sample breaks

second experiment… set confining pressure higher and increase axial load (stress) until new sample breaks

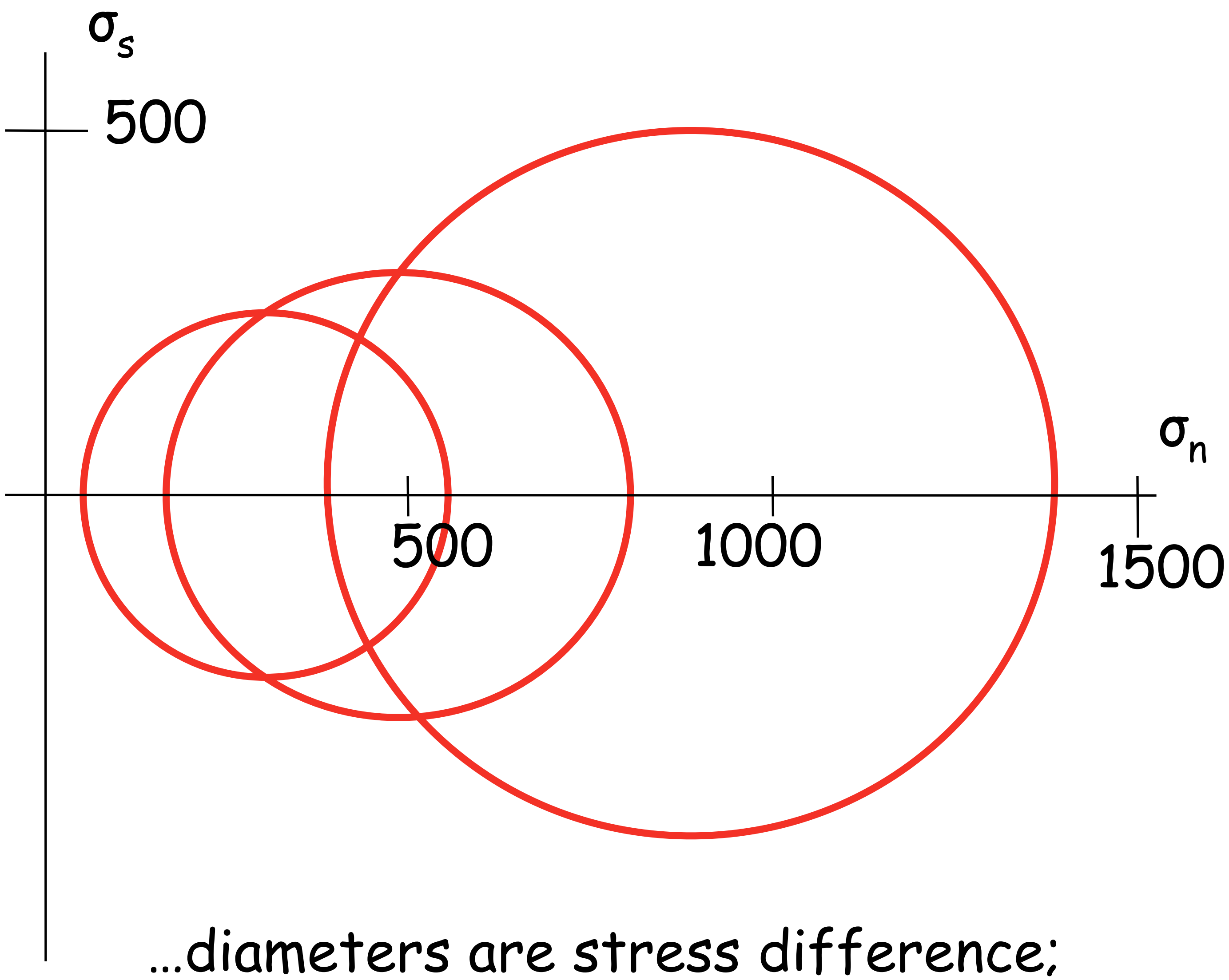
keep repeating experiments…

you will generate a series of pairs of confining stresses and associated axial stresses at which samples break...

σ_c	σ_a	$\sigma_a - \sigma_c$
40	540	500 MPa
150	800	650 MPa
400	1400	1000 MPa

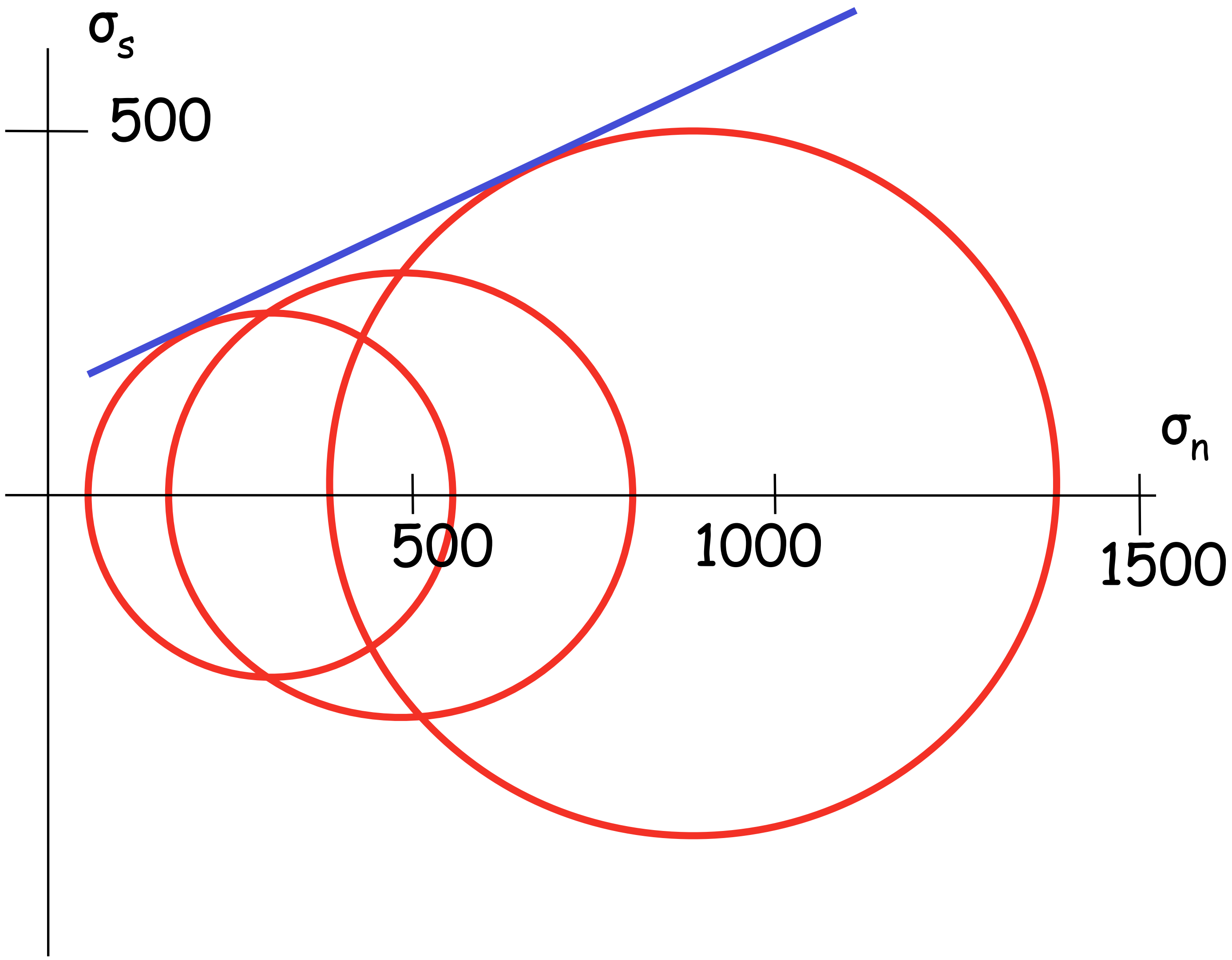
we use these as σ_1 and σ_3 and plot Mohr circles...

get a sequence of circles offset from one another



...diameters are stress difference;
...centers are stress sum/2

Failure envelope



Mohr circles that define
stress states where
samples fracture
(critical stress states)
together define the
failure envelope
for a particular rock

failure envelope is tangent to circles of all critical stress states and is a straight line…

can also draw failure envelope in negative quadrant for σ_s (mirror image about σ_n axis)

Coulomb criterion

what does this straight line mean?

…corresponds to **Coulomb fracture criterion**

Charles Coulomb in 18th century proposed that formation of shear stress parallel to failure relates to normal stress by…

$$\sigma_s = C + \tan\varphi\,(\sigma_n) \quad \text{(empirical)}$$

σ_s = shear stress parallel to fracture at failure

C = cohesion of rock (constant)

σ_n = normal stress across shear zone at instant of failure

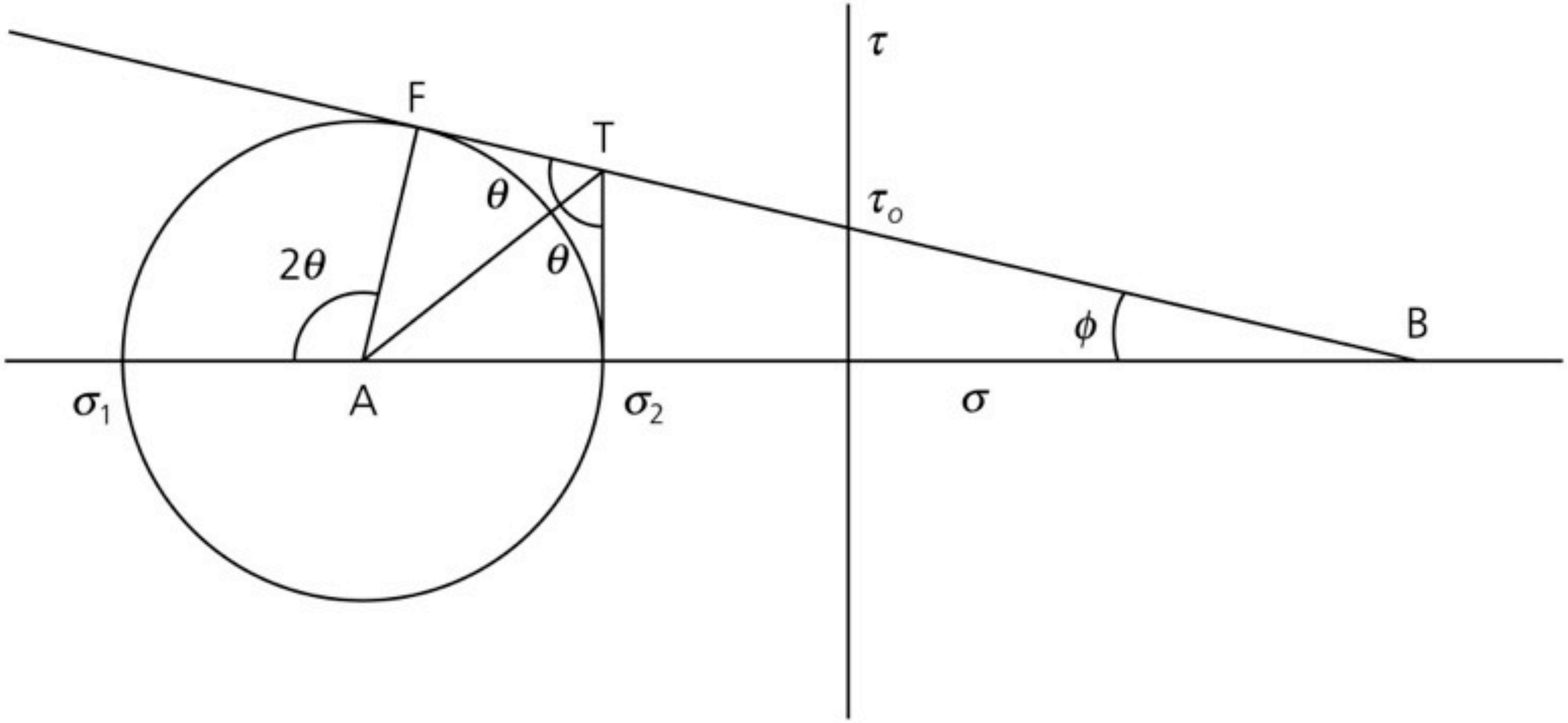
$\tan\varphi = \mu$ = coefficient of internal friction (constant of proportionality)

this has form of $y = mx + b$ (equation of a line)

$y = \sigma_s$ $x = \sigma_n$ b = intercept on σ_s axis m = slope = $\tan\varphi = \mu$

so Coulomb criterion plots as straight line on σ_n , σ_s plot

Figure 5.7-7: Relation between cohesive stress, internal friction, and geometry of failure.



Laboratory experiments on rocks under compression show that fracture occurs when a critical combination of the absolute value of shear stress and the normal stress is exceeded.

Coulomb-Mohr failure criterion: $|\tau| = \tau_o - n\sigma$

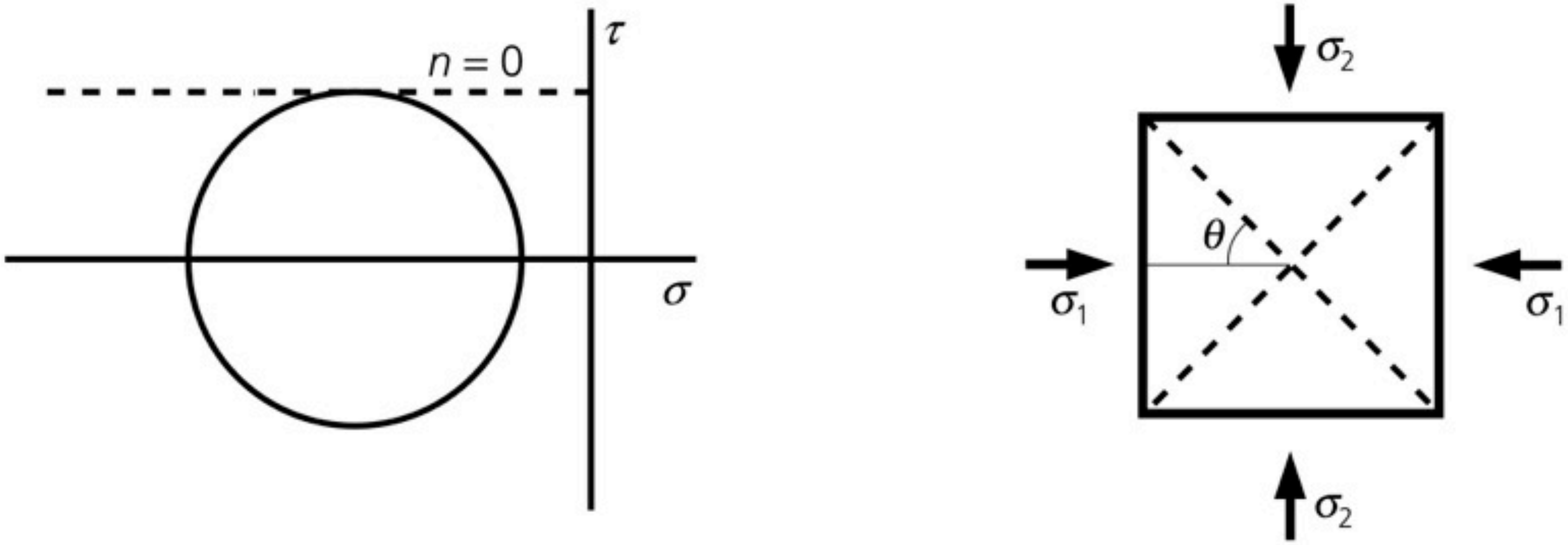
τ_o = cohesive strength; n = coefficient of internal friction

$|\tau| = \tau_o - \sigma \tan\phi$ where $n = \tan\phi$ and ϕ

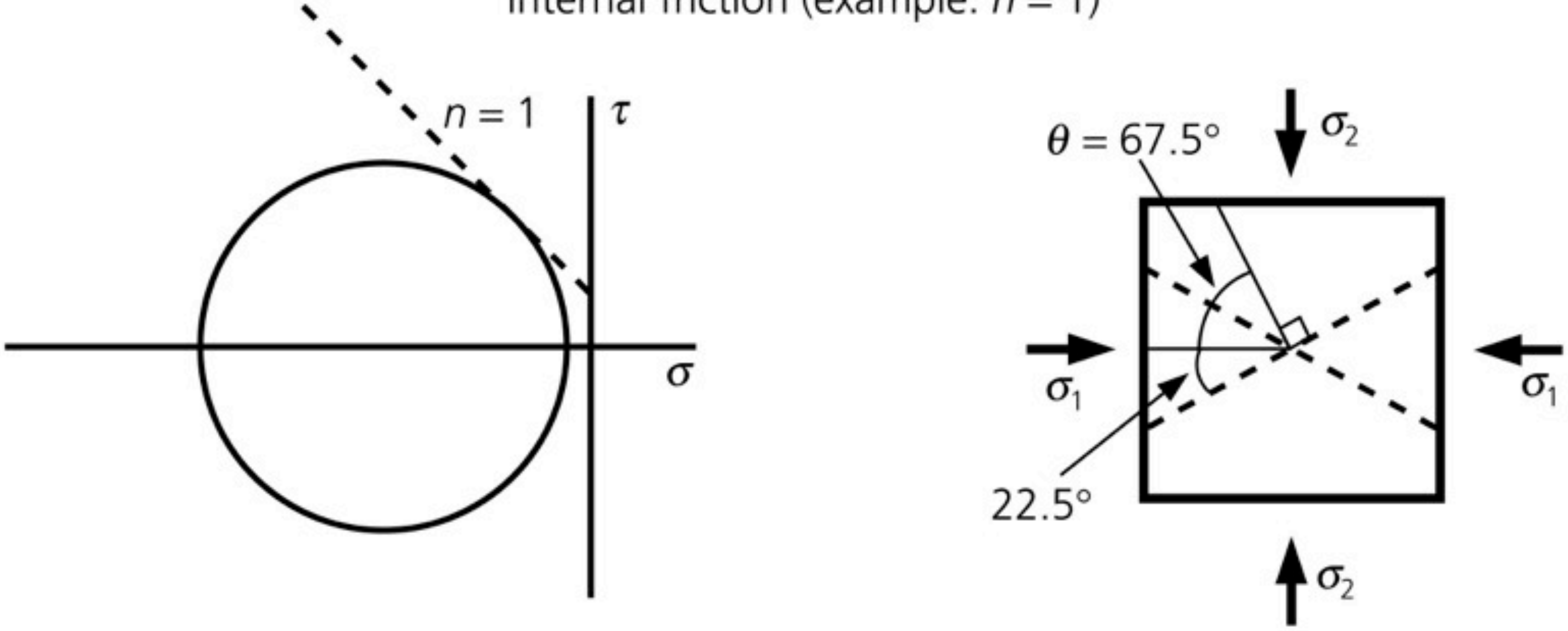
$\phi = 2\theta - 90^\circ$ so $\theta = \phi/2 + 45^\circ$

Figure 5.7-8: Failure with and without internal friction.

No internal friction

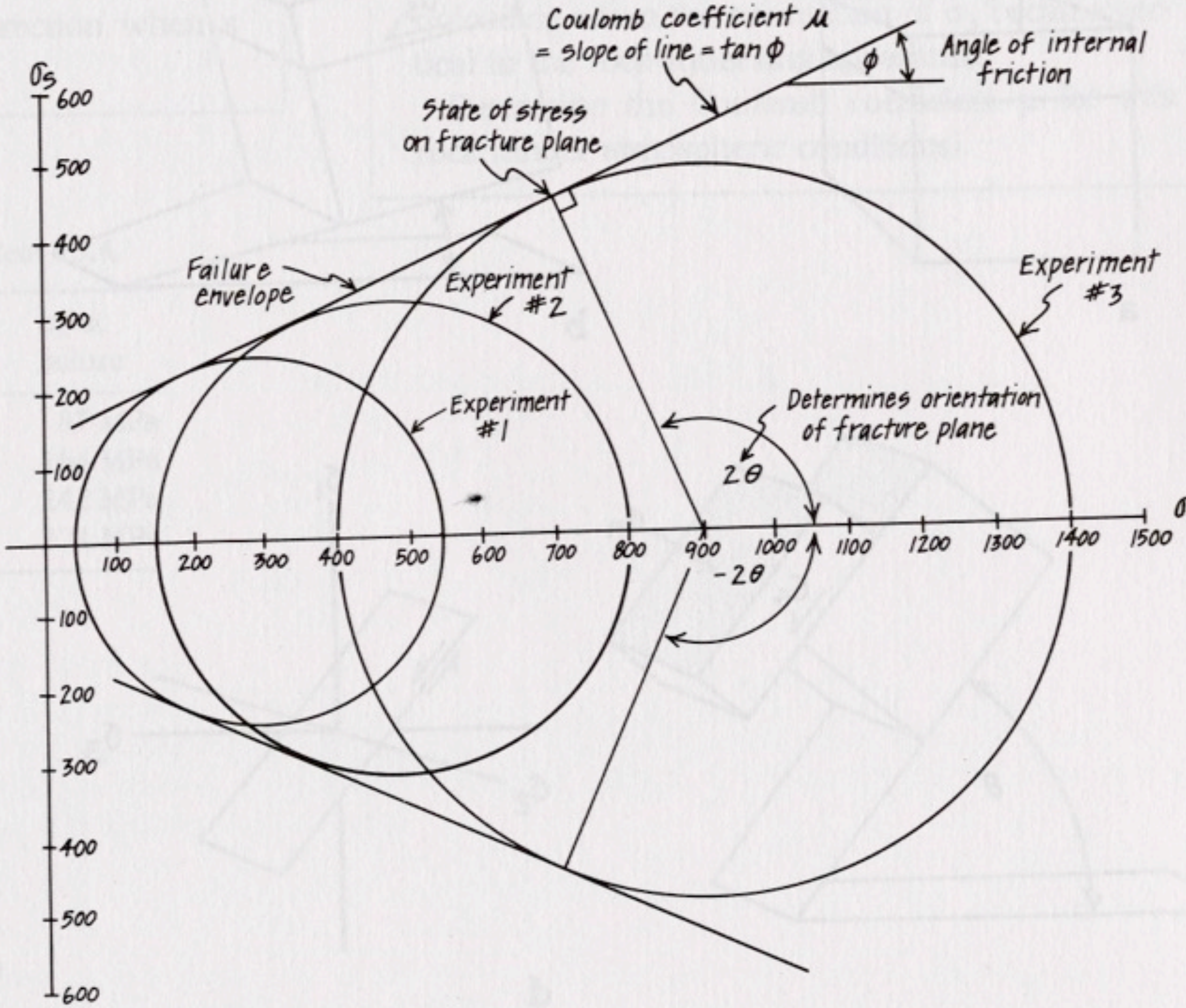


Internal friction (example: $n = 1$)



Mohr circle & Coulomb criterion

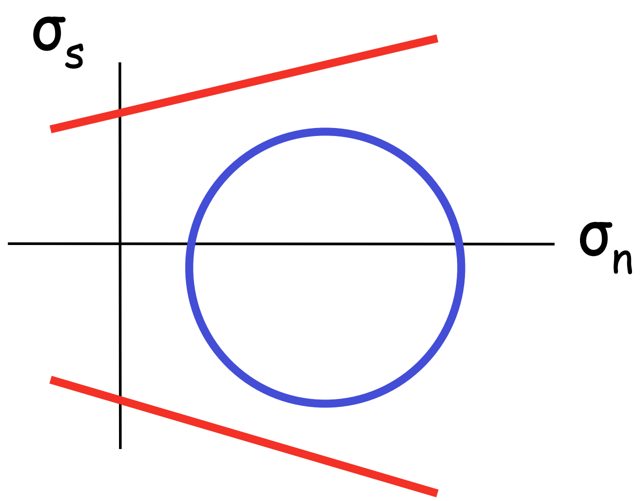
return to our Mohr circle with Coulomb criterion plotted



from: Rowland and Duebendorfer, 1994

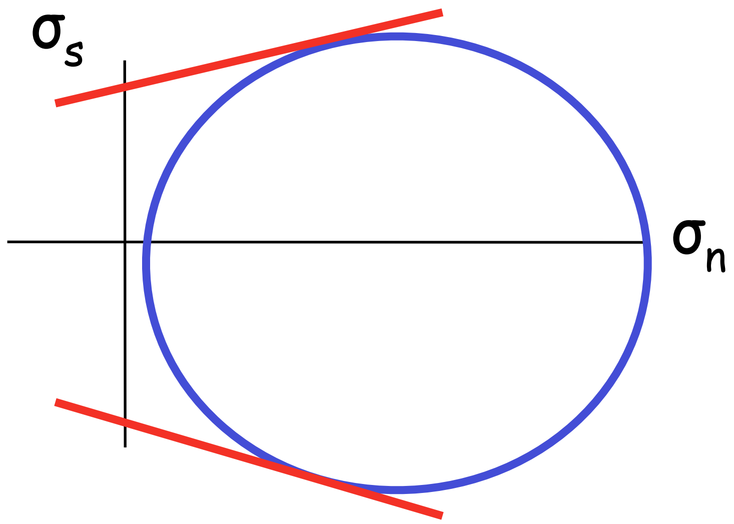
orientation of planes that break is specified by 2θ of point that is both on circle and line plot of either Coulomb or Mohr-Coulomb criterion defines failure envelope on Mohr diagram

failure envelope separates fields of “stable” and “unstable” stress



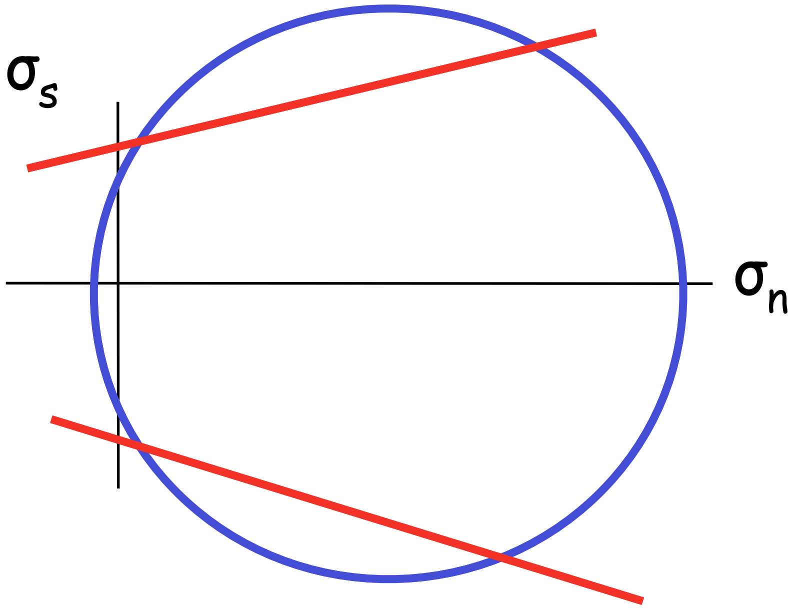
no failure
(stable)

Mohr circle
inside envelope



failure at two points
(brittle failure)

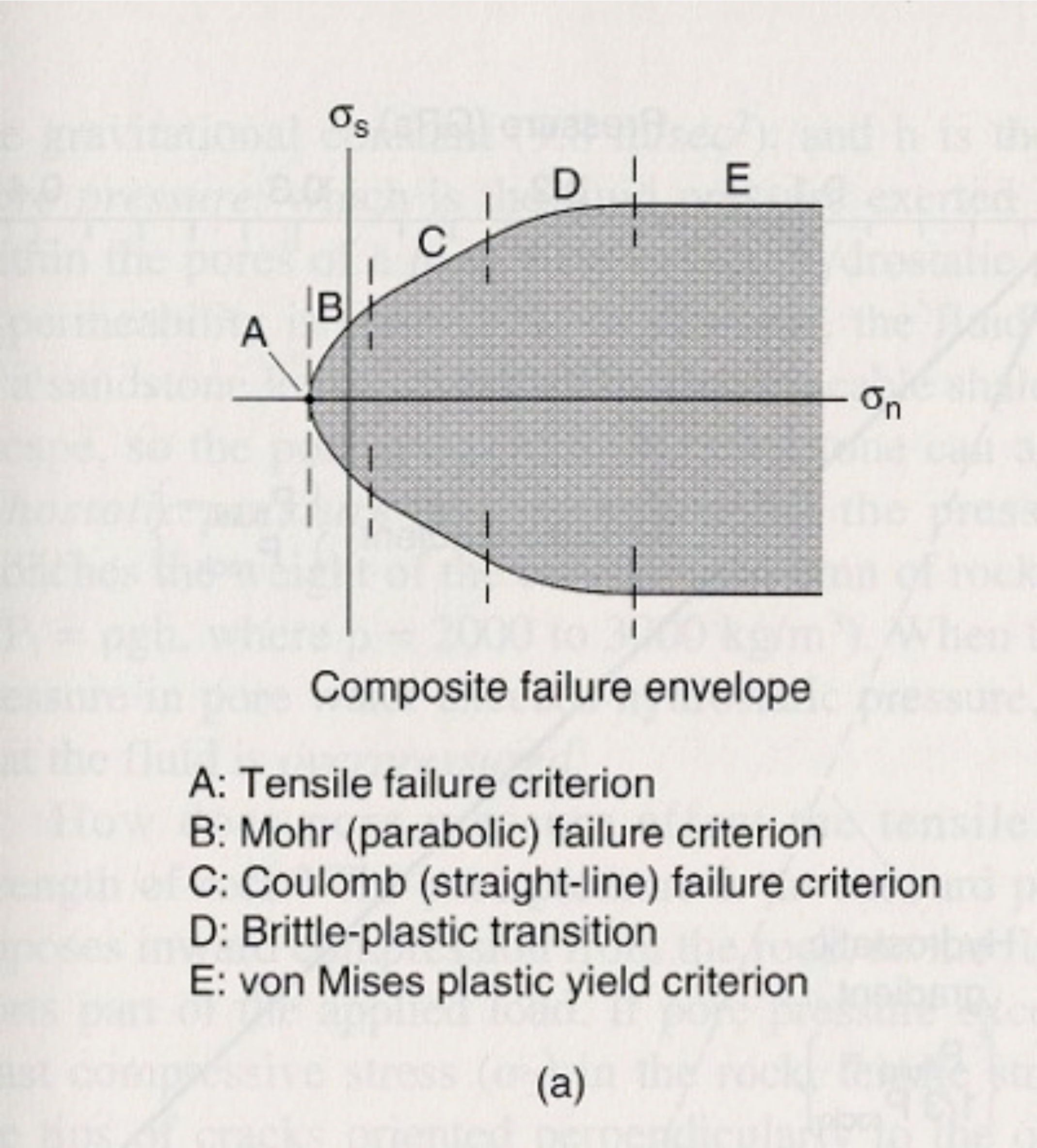
Mohr circle
tangent to envelope



impossible
(unstable)

Mohr circle
outside envelope

can create composite failure envelope from empirical criteria

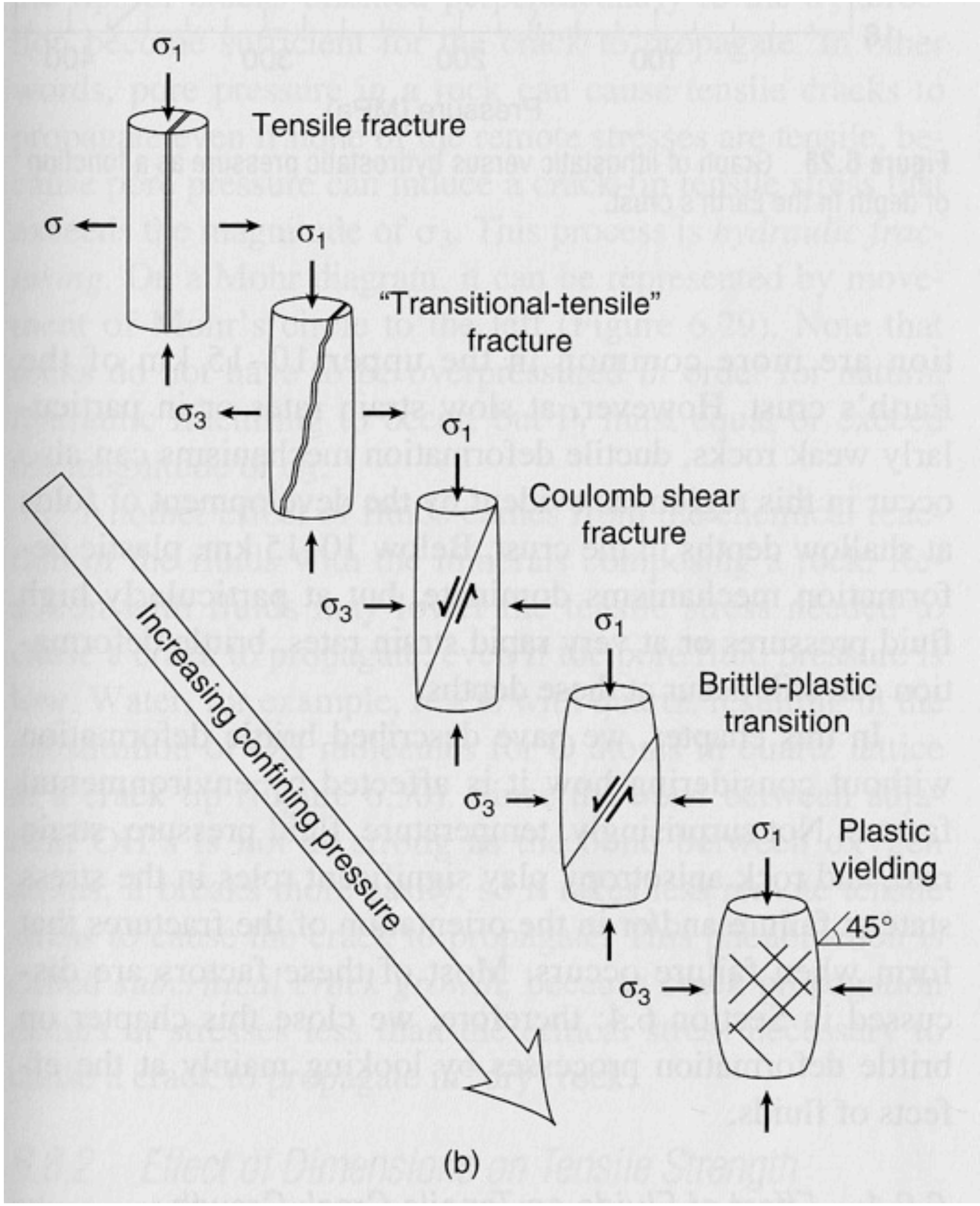


Composite failure envelope

A: Tensile failure criterion
B: Mohr (parabolic) failure criterion
C: Coulomb (straight-line) failure criterion
D: Brittle-plastic transition
E: von Mises plastic yield criterion

(a)

types of fracture (right) for composite curve (above)

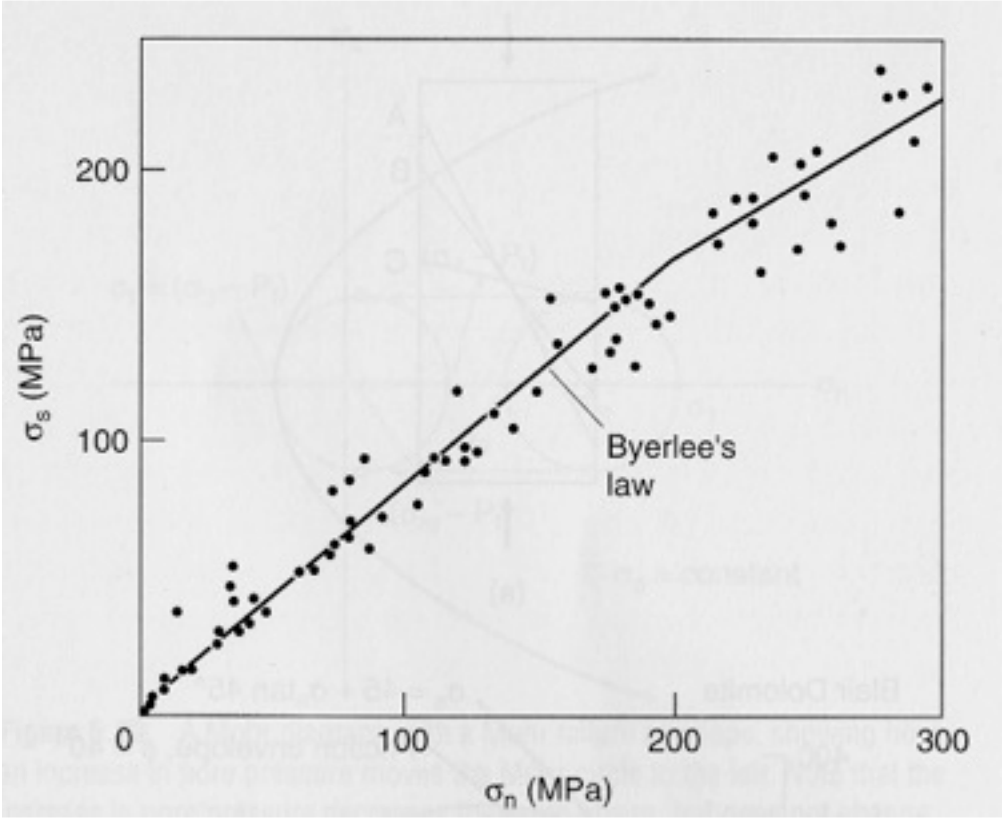


(b)

both from: van der Pluijm and Marshak, 1997

3) Frictional sliding

friction requires certain critical shear stress to be reached
before sliding initiates on preexisting fracture



failure criterion for frictional sliding

experimental data show that this plots as
sloping straight line on Mohr diagram
failure criterion for frictional sliding
is largely independent of rock type
σ_s / σ_n = constant
Byerlee's law

from: van der Pluijm and Marshak, 1997

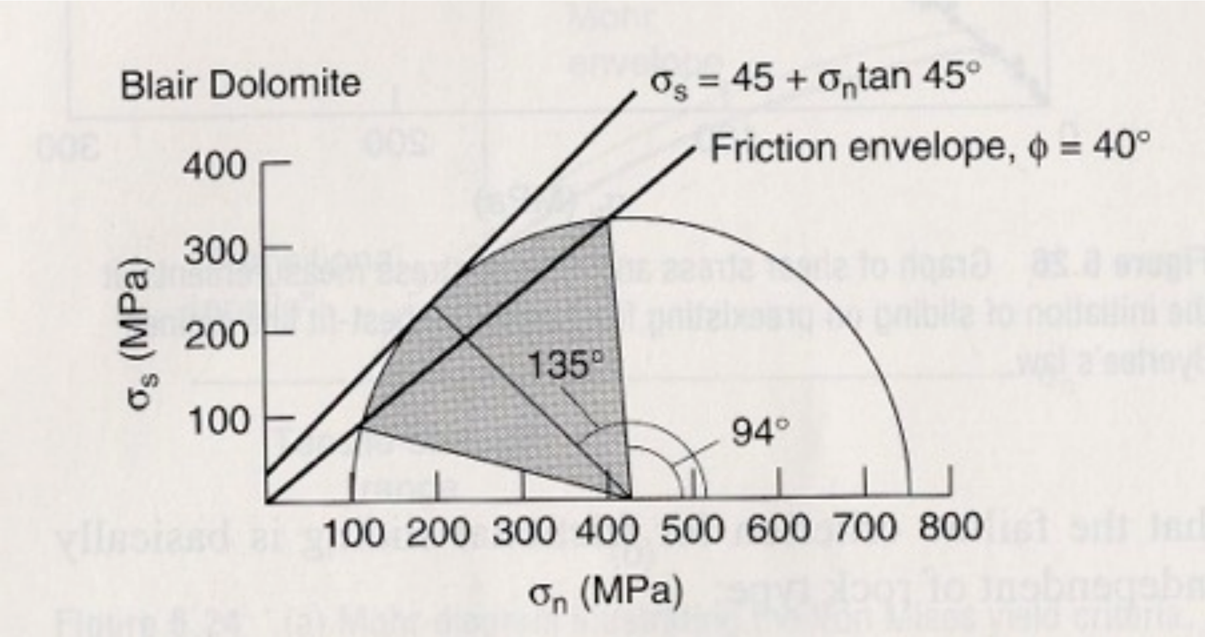
for $\sigma_n < 200$ MPa: $\sigma_s = 0.85\ \sigma_n$
for 200 MPa $< \sigma_n <$ 2000 MPa: $\sigma_s = 50$ MPa $+ 0.6\ \sigma_n$

the important question…

will new fractures form or will existing fractures slide?

examine failure envelopes to decide

figure below shows both Byerlee's law for frictional sliding and Coulomb shear fracture envelope for Blair Dolomite



from: van der Pluijm and Marshak, 1997

slope and intercept of two envelopes are different…
for specific orientations of preexisting fractures,
Mohr circle touches frictional envelope first

→ **preexisting fractures will slide before new fracture forms**

Earthquake rupture

Can be described by: formation and b) propagation of a crack.

The crack tip acts as a stress concentrator and if the stress exceeds some critical value then sudden slip occurs, and it drops to the dynamic frictional value; when the slip has stopped the stress reaches a final level

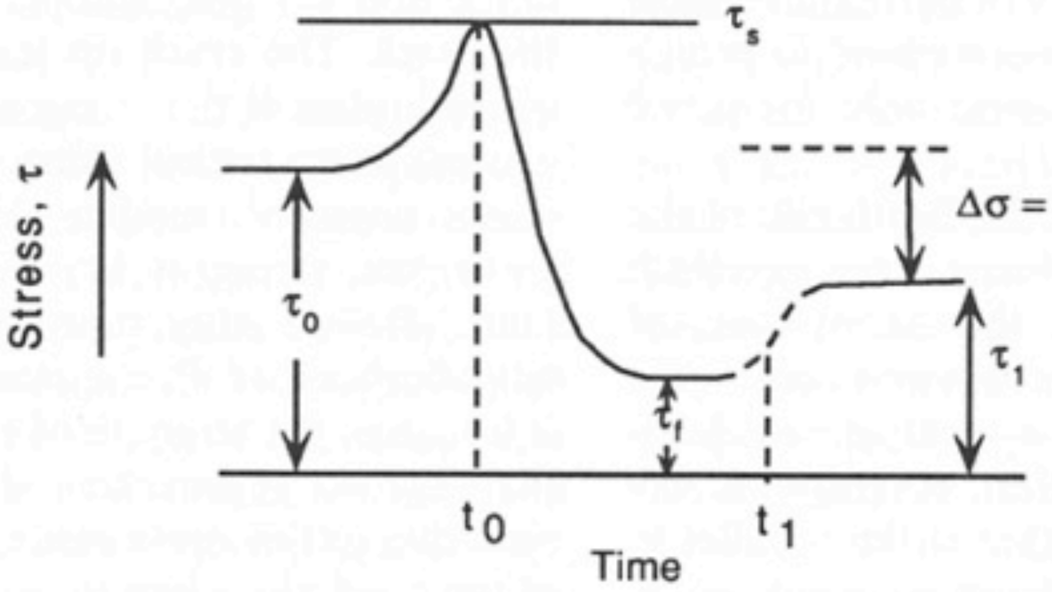
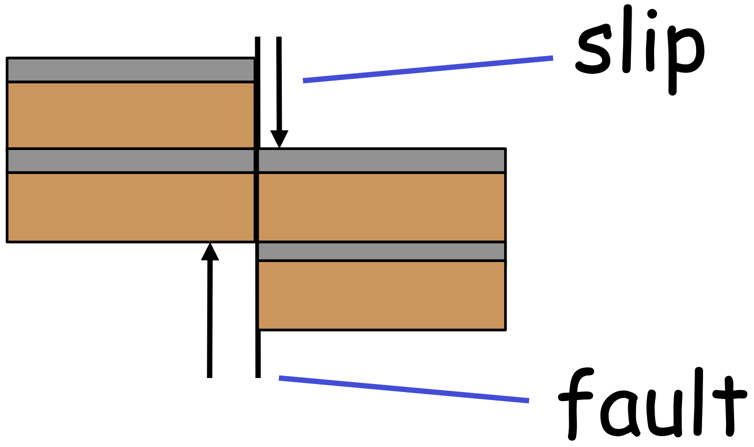


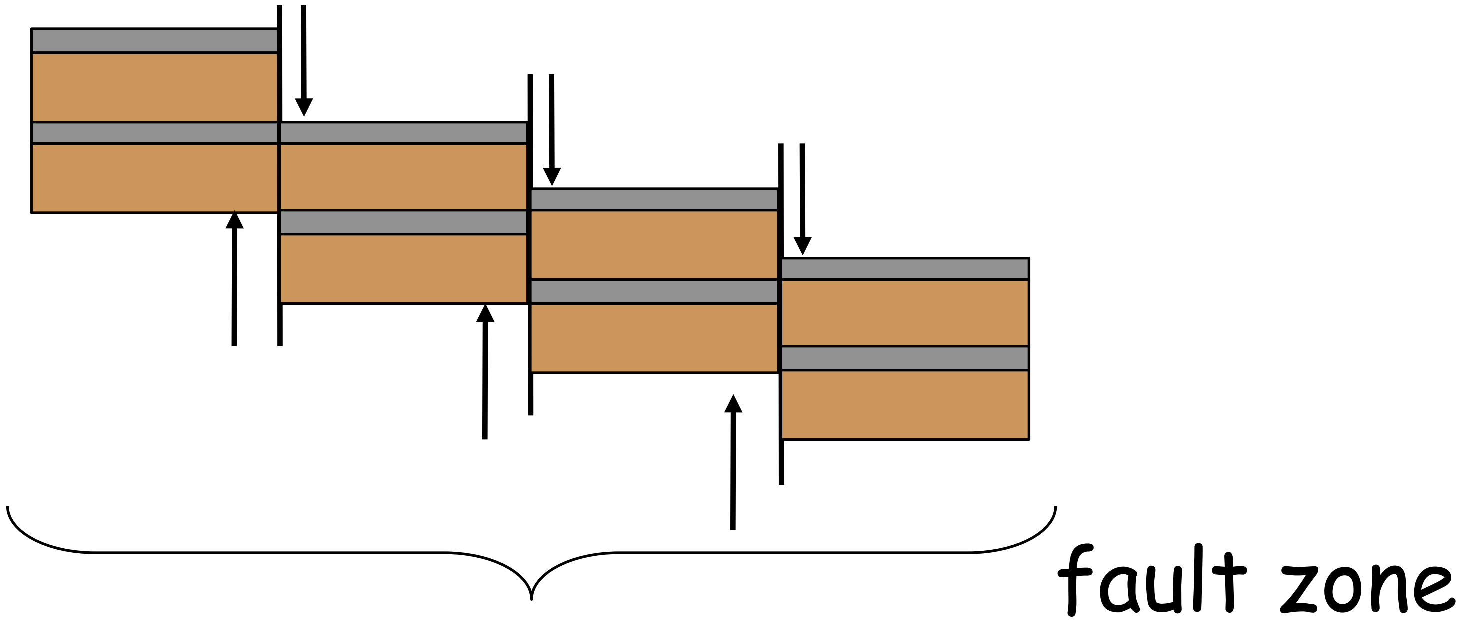
FIGURE 9.2 Stress at a point on a fault surface. As the rupture front approaches the point, stress increases to a value of τ_s, after which failure occurs at the point. The point slips to a displacement D, and stress is reduced to some value τ_f. The difference between the initial stress and the final stress, $\Delta\sigma$, is defined as the stress drop. (After Yamashita, 1976.)

Faults

fault:
- surface across which measurable **slip** occurs;
- slip is **parallel** to the fault surface (**shear displacement**);
- slip develops primarily by **brittle** processes--
  distinguishes faults from shear zones



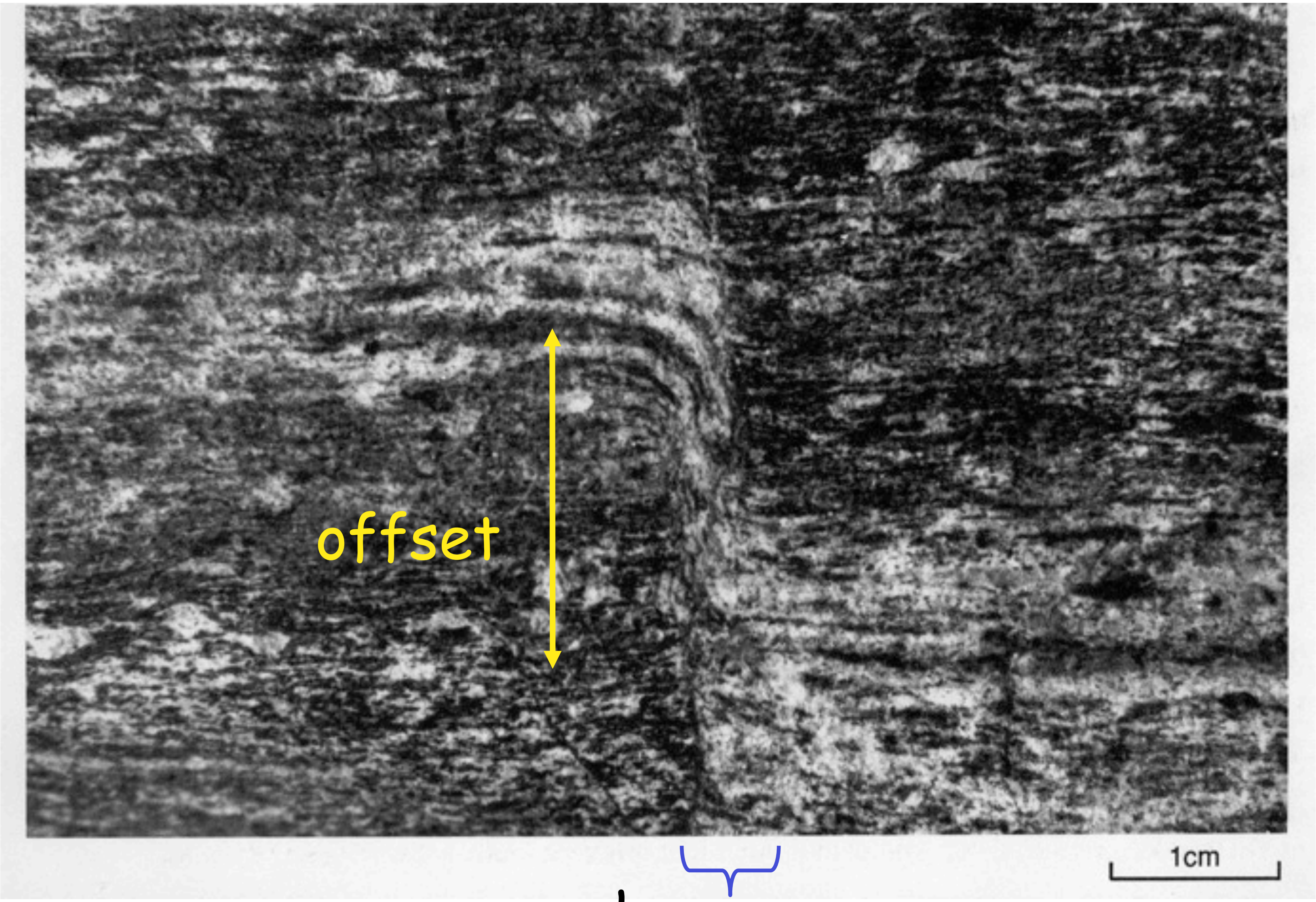
fault zones: brittle structures where loss of cohesion and slip occur on several faults within a band of definable width



Shear zones

shear zones: **ductile** structures

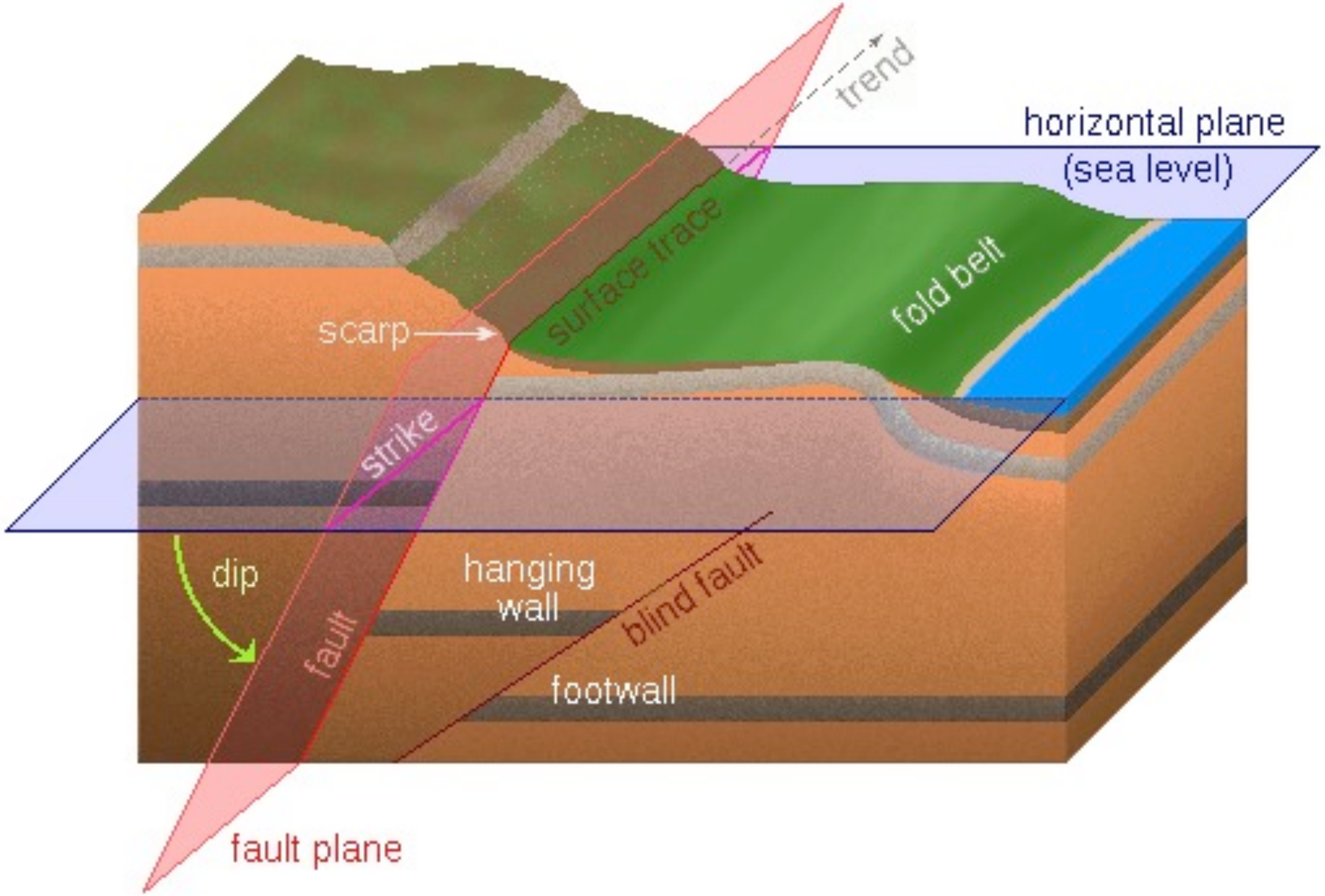
- rock does not lose mesoscopic cohesion
- form at deep crustal levels;
- deformation is distributed across band of definable width;



shear zone

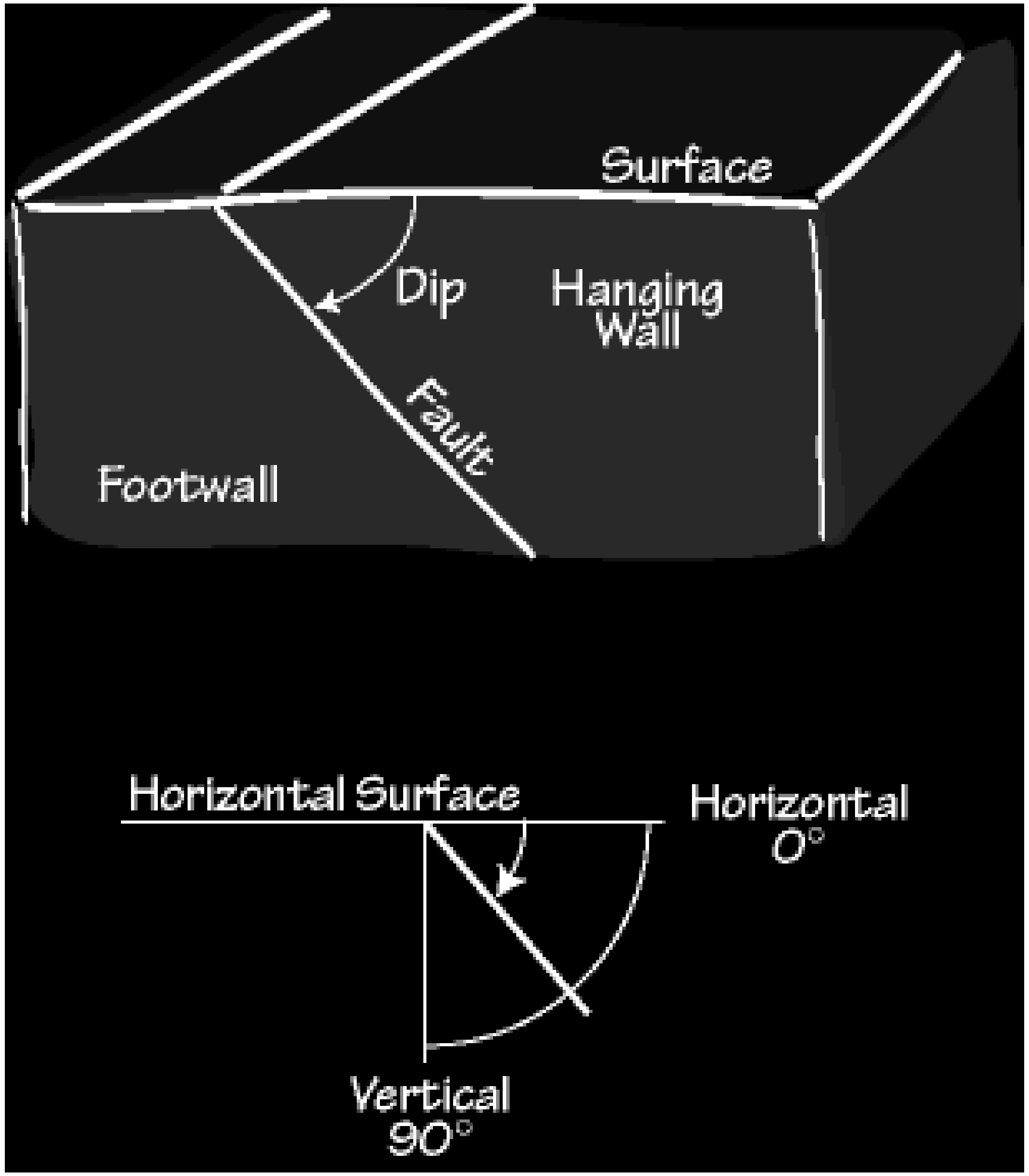
Earthquakes and Faults

- Earthquakes occur on faults, but not all of the fault ruptures during each earthquake.



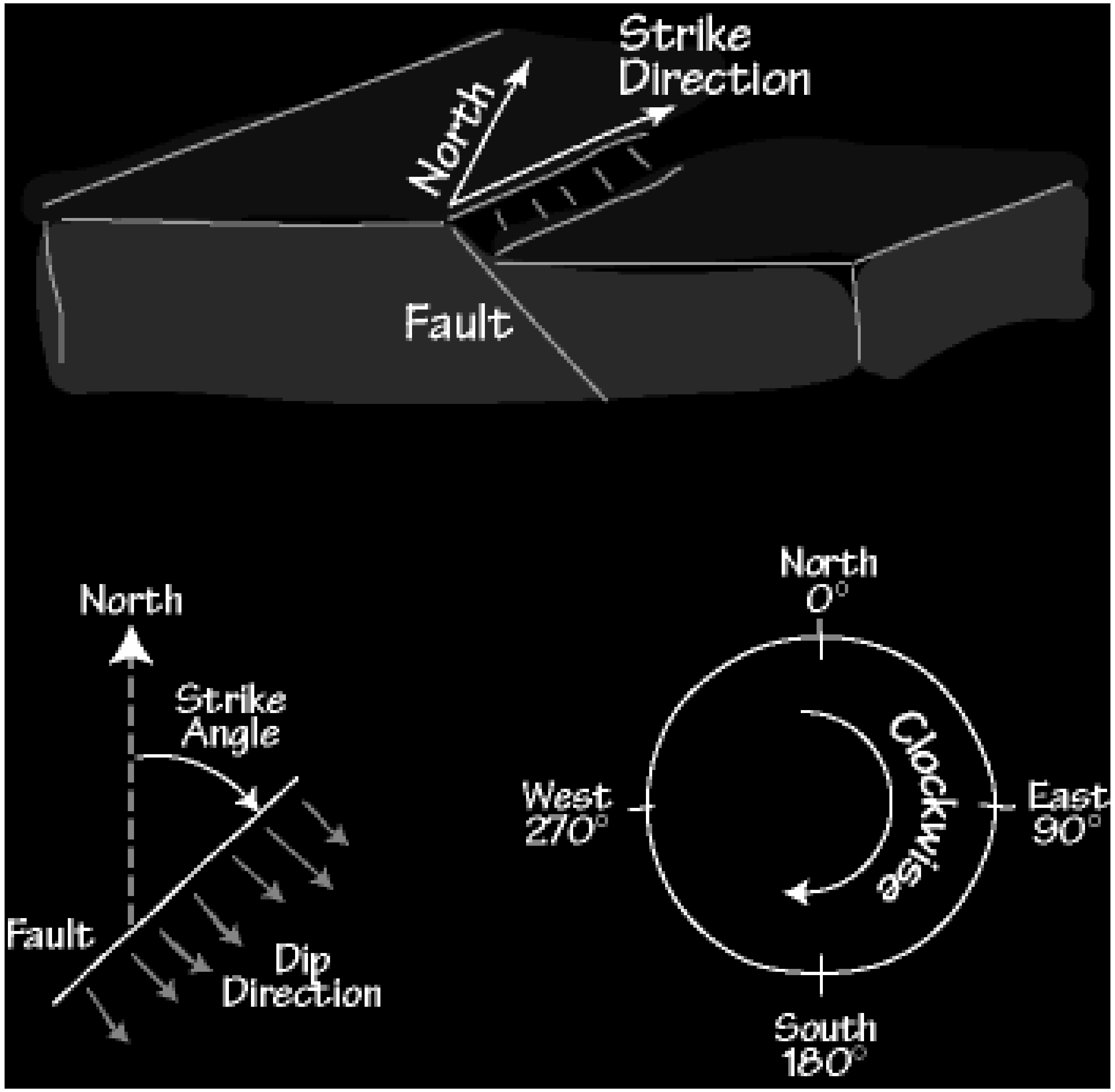
Fault Geometry Terminology: DIP

The orientation of the fault surface with respect to Earth's surface is defined by the fault dip.



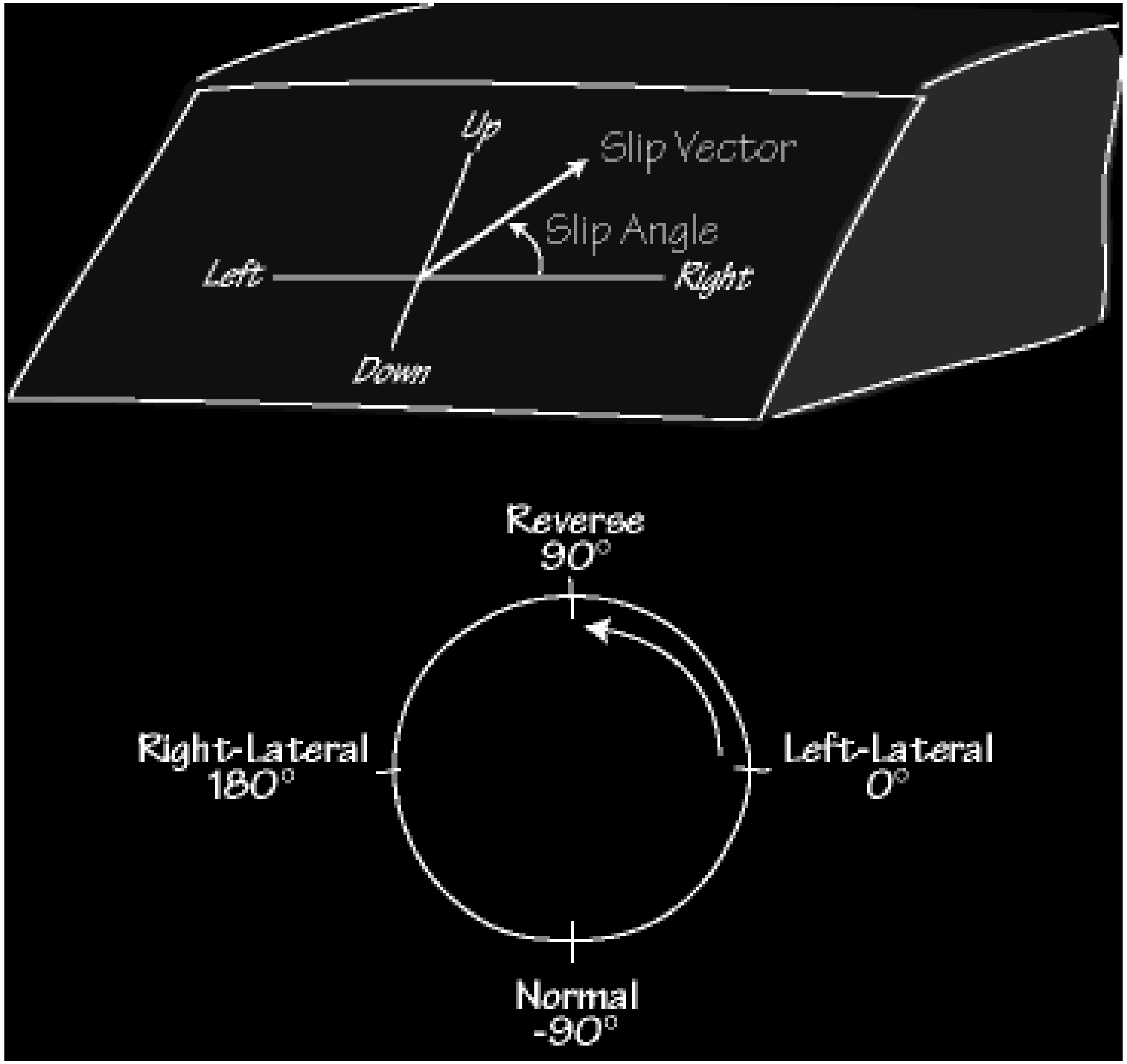
Fault Geometry Terminology: STRIKE

Strike is an angle use to describe the orientation of the fault surface with respect to North.



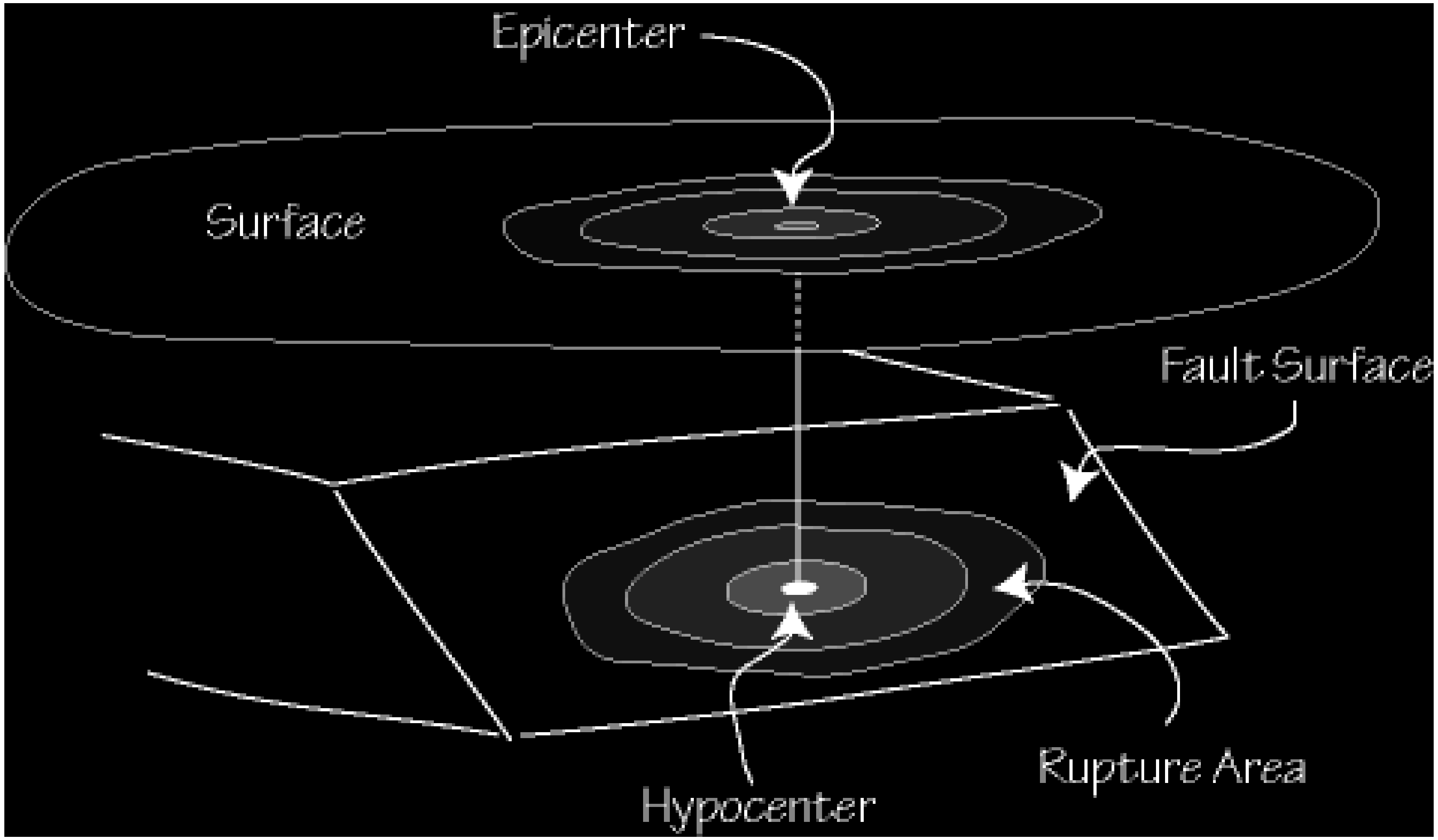
Fault Geometry Terminology: SLIP

Slip is the angle used to describe the orientation of the movement of the hanging wall relative to the foot wall.

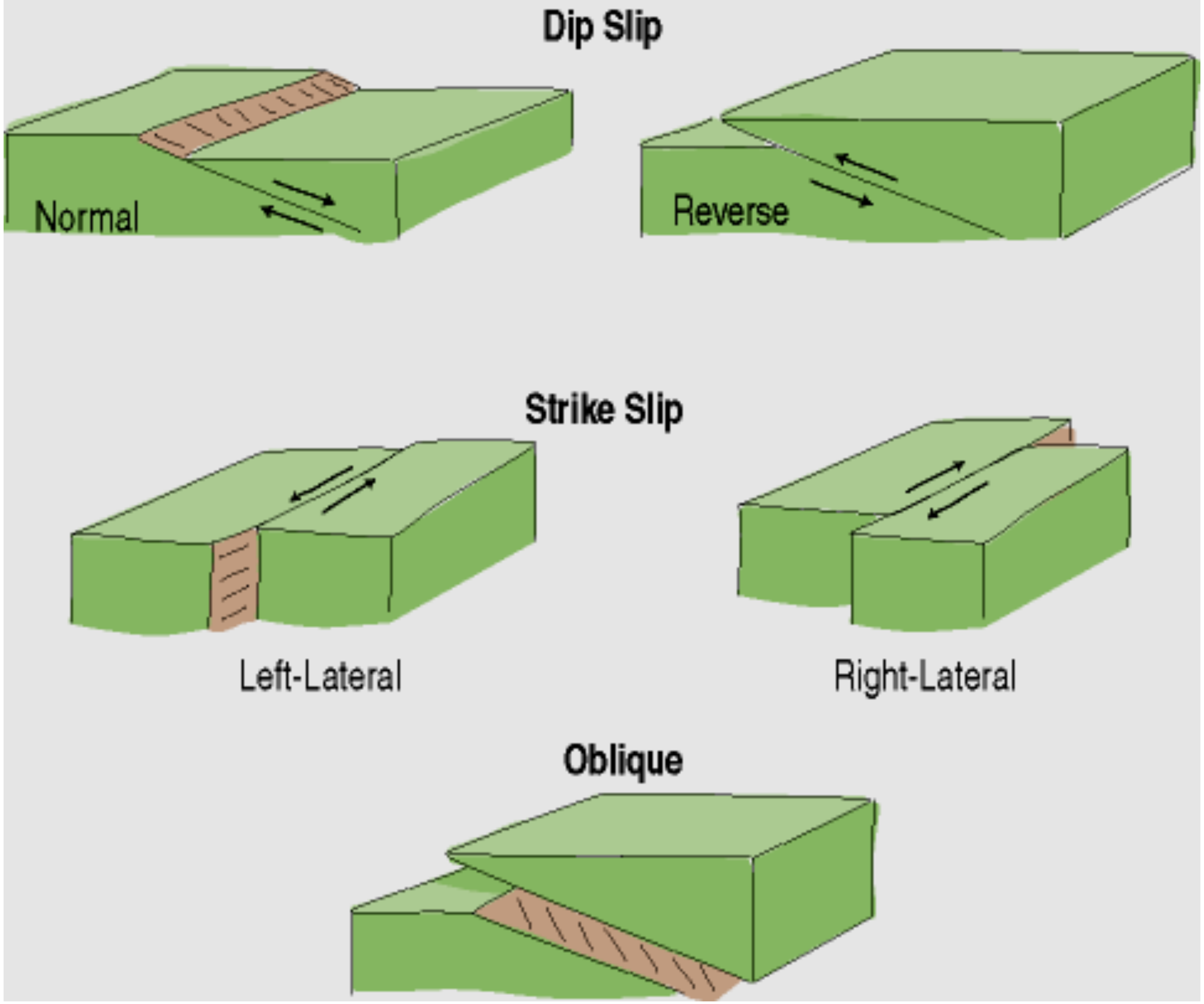


Hypocenter and Epicenter

- The hypocenter (or focus) is the place where the rupture begins, the epicenter is the place directly above the hypocenter.



Faulting Summary



Faults which move along the direction of the dip and are described as either **normal** or **reverse**, depending on their motion.

The hanging wall slips horizontally (no motion in the direction of fault dip).

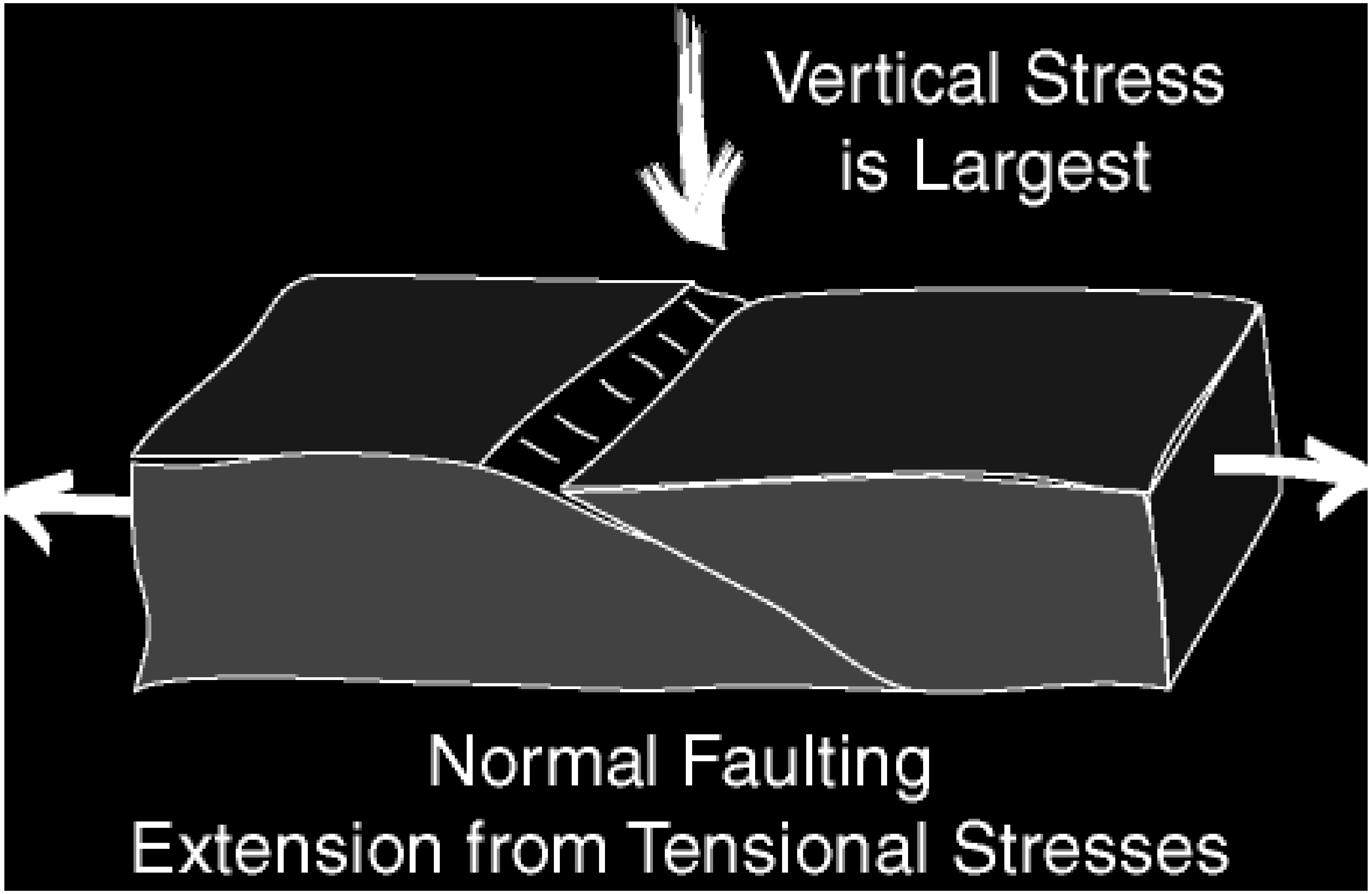
There are 2 cases depending on how the rocks on the other side of the fault move - **right lateral** and **left lateral**.

Faulting and Stresses

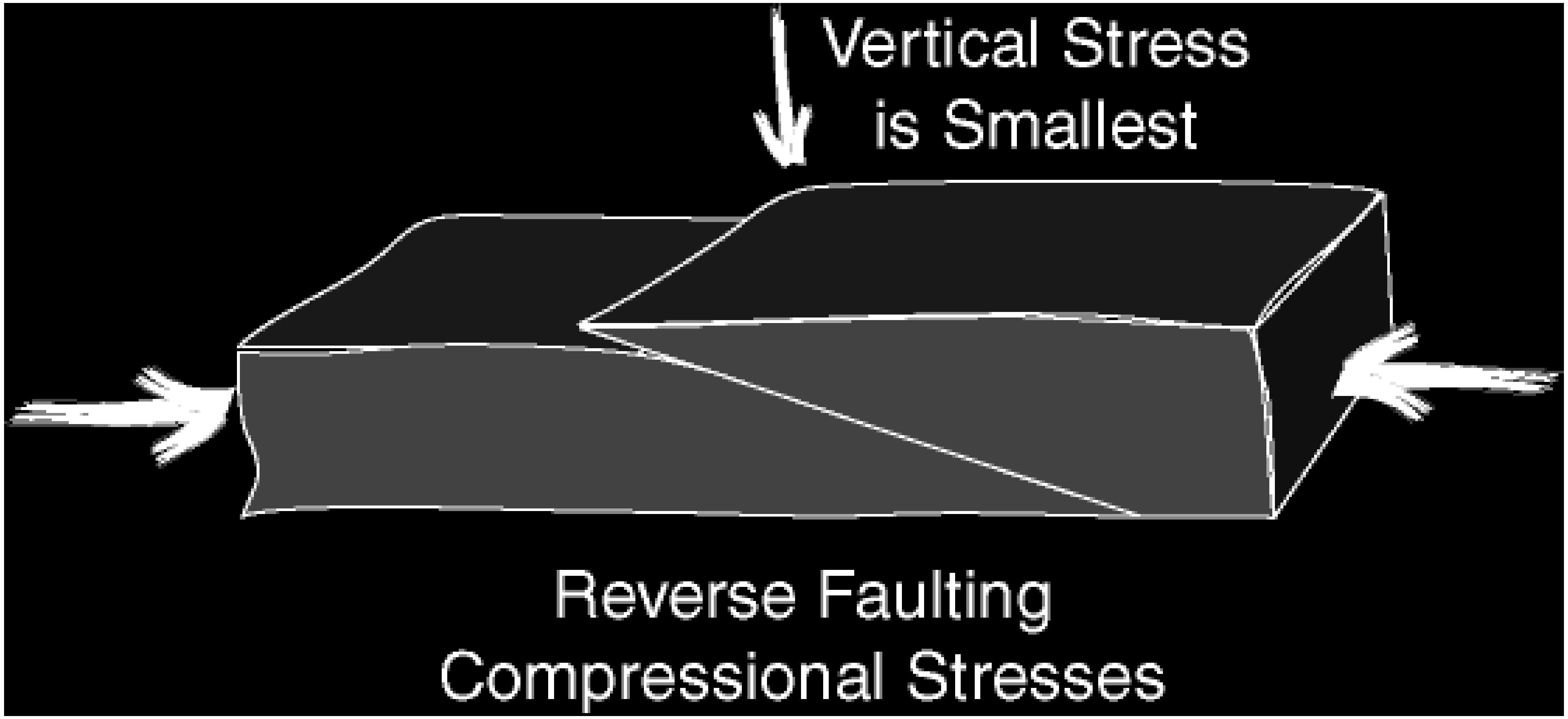
- The style of faulting (normal, reverse, etc.) also tells us about the stresses acting within Earth.

- We describe the stresses by considering three stresses, two horizontal and the vertical.

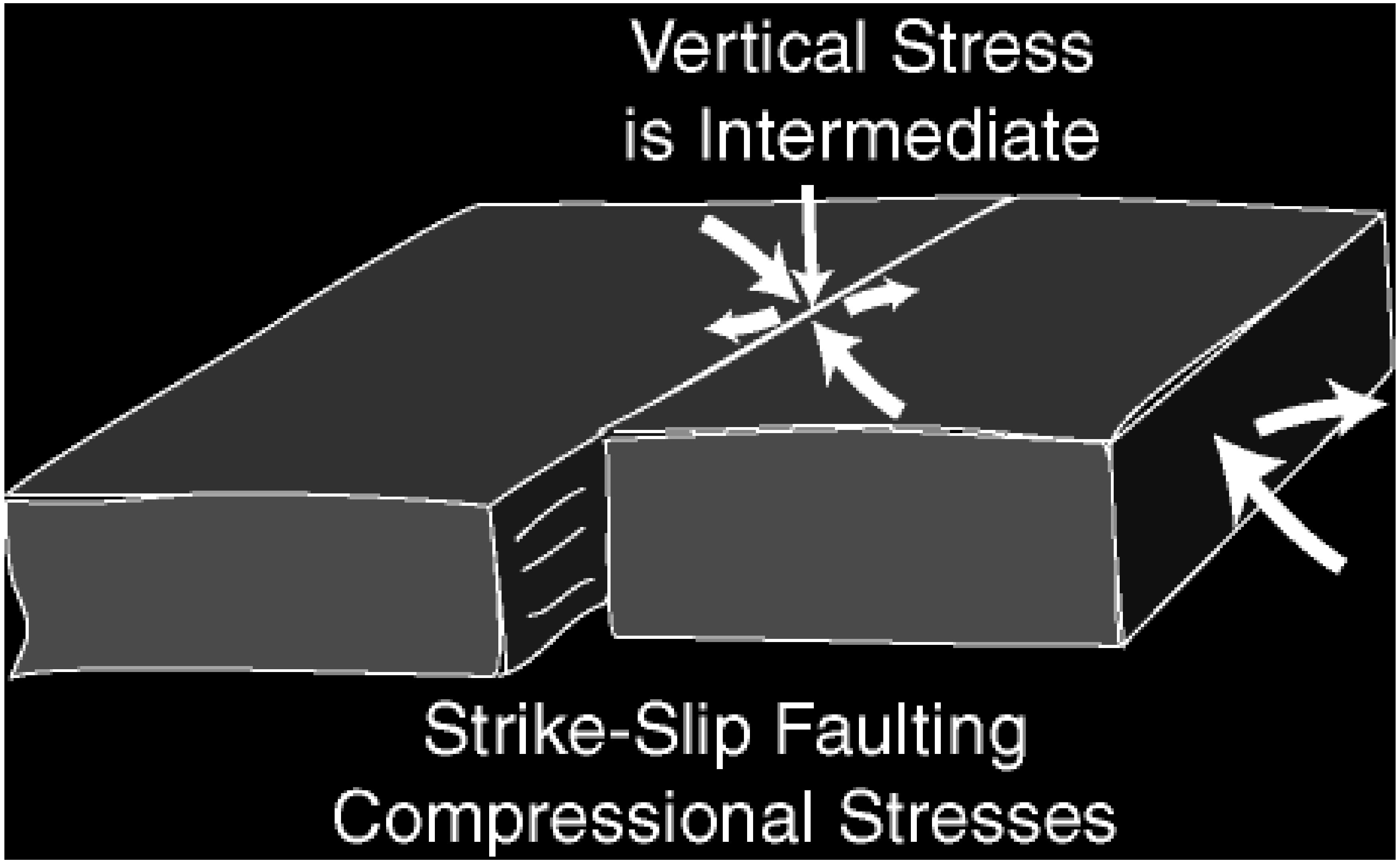
Normal Faulting Stresses



Reverse Faulting Stresses



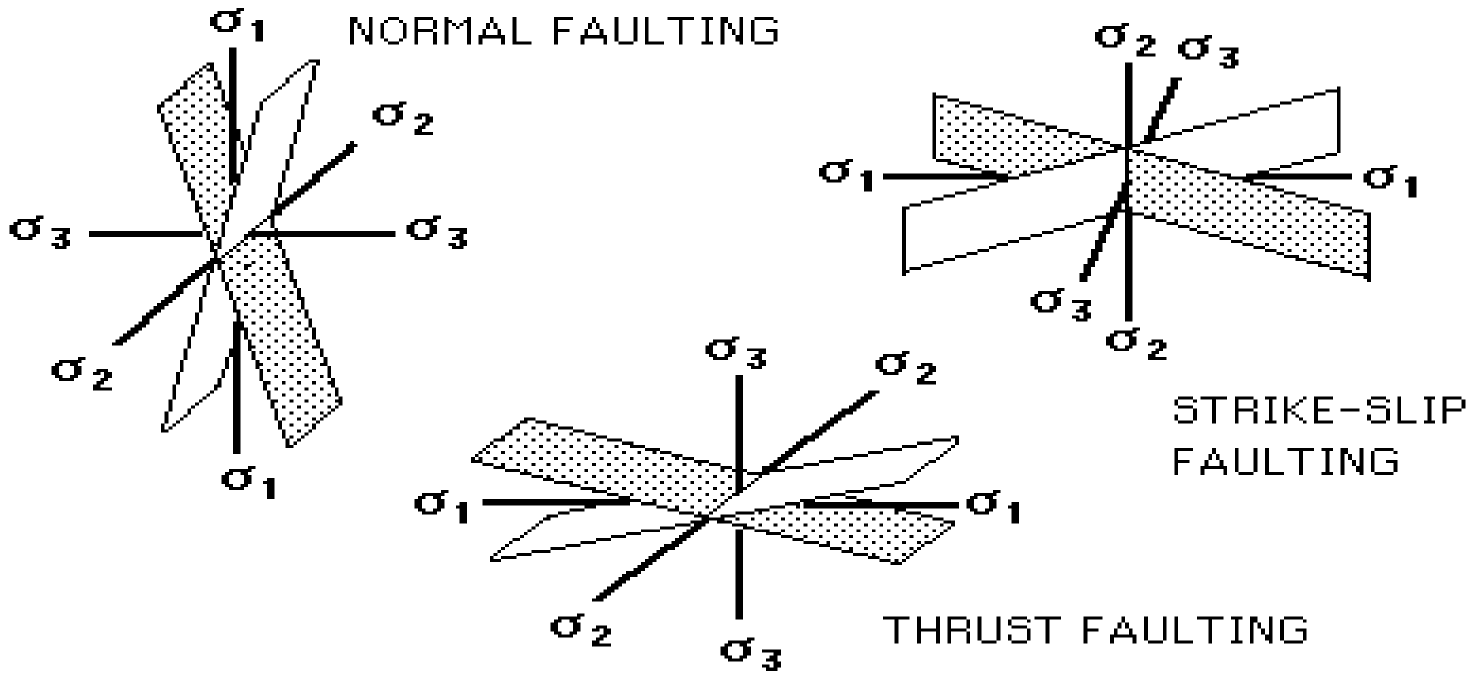
Strike-Slip Faulting Stresses

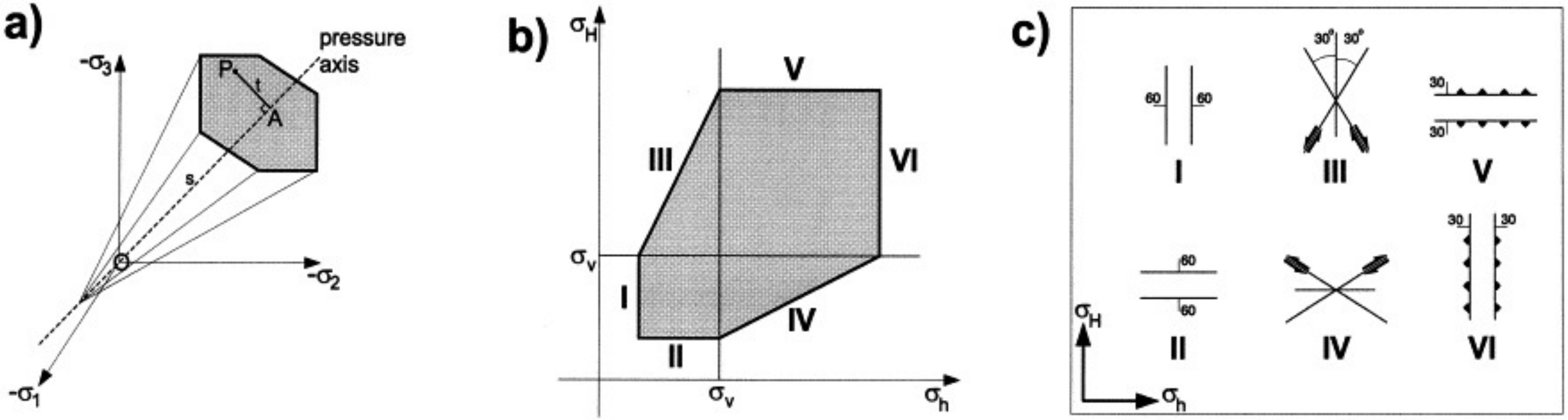


Friction

A number of factors can control friction: temperature, slip rate and slip history. Many materials become weaker with repeated slip (slip weakening).
They may exhibit an inverse dependence of friction on slip velocity (velocity weakening).
Stick slip behaviour is observed only at temperatures below 300°C.

Anderson's theory of faulting: he recognized that principal stress orientations could vary among geological provinces within the upper crust of the earth. He deduced the connection between three common fault types: normal, strike-slip, and thrust and the three principal stress systems arising as a consequence of the assumption that one principal stress must be normal to the Earth's surface.





(a) In principal stress space $(\sigma_1,\sigma_2,\sigma_3)$, the Mohr-Coulomb failure criterion takes the form of an irregular hexagonal cone around the space diagonal (the pressure axis).

(b) A cross section through a Mohr-Coulomb failure surface at σ_v=constant (assuming Andersonian stress conditions). Each side of the resulting figure defines the relations between the three principal stresses. Dependent on whether σ_v is the largest, intermediate or smallest principal stress, a different class of faulting/ fracturing can be identified.

(c) Shows these different classes in map view (assuming a friction angle ϕ=30° , which is typical for most rocks).