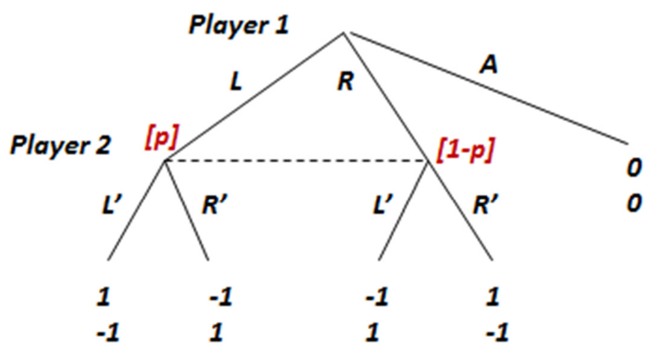
Example 4



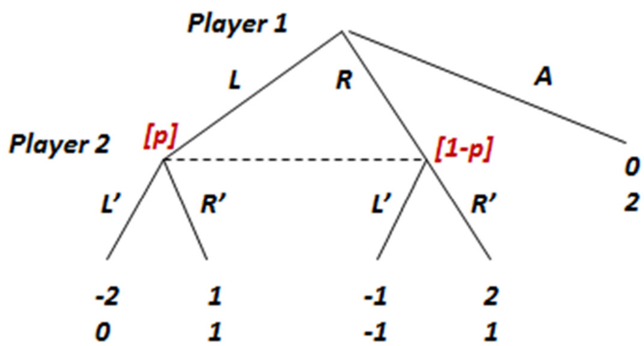
Try to check if the following strategy profiles are PBE

1) $s_1 = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ $s_2 = \left(\frac{1}{2}, \frac{1}{2}\right)$
2) $s_1 = (0, 0, 1)$ $s_2 = \left(\frac{1}{2}, \frac{1}{2}\right)$

Solution

1) The Player 2's expected value from playing L' is $E(L') = -p + 1 - p = 1 - 2p$ and that from playing R' is $E(R') = p - (1 - p) = 2p - 1$. In order to play a mixed strategy player 2 needs that $E(L') = E(R')$ that rewritten is $1 - 2p = 2p - 1$. It is satisfied only when $p = 0.5$. The information set is reached with positive probability, then player 2's beliefs are to be consistent with the strategy of player 1. Applying Bayes rule we find that (when the information set of player 2 is reached) the probability that player 1 played L is $\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}} = \frac{1}{2}$. Then $p = \frac{1}{2}$ is consistent with the strategy profile of player 1. Now we have to check if the strategy of player 1 is optimal (given the strategy of player 2). Given that player 1 plays a mixed strategy the requirement is that the expected values from the actions played with strictly positive probability are equal. It is easy to check that this condition is satisfied: $E(A) = E(R) = E(L) = 0$
2) The Player 2's expected value from playing L' is $E(L') = -p + 1 - p = 1 - 2p$ and that from playing R' is $E(R') = p - (1 - p) = 2p - 1$. In order to play a mixed strategy player 2 needs that $E(L') = E(R')$ that rewritten is $1 - 2p = 2p - 1$. It is satisfied only when $p = 0.5$. Note that in this case the player 2's information set is reached by probability 0. Then the beliefs on p can be arbitrary. Then we don't have to check consistency of beliefs with the strategy of player 1. We have to check if the strategy of player 1 is optimal (given the strategy of player 2). The condition to check if $E(A) \geq E(R) = E(L)$. It ieasy to check that it is satisfied by equality.

Example 5



Find all Nash equilibria and check they are PBE

2

Solution

Nash equilibria

		Player 2	
		L'	R'
Player 1	L	-2, 0	1, 1
	R	-1, -1	2, 1
	A	0, 2	0, 2

Two Nash equilibria:

a) $\{(A),(L')\}$

b) $\{(R),(R')\}$

Consider $\{(A),(L')\}$ and check if it is PBE. For Player 2 to play L' never is optimal, i.e. for all possible beliefs p for player 2 is optimal to play R'. Then this NE is not PBE.

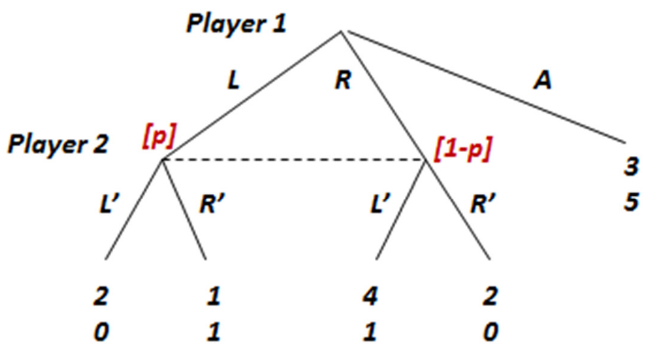
Consider $\{(R),(R')\}$ and check if it is PBE. For Player 2 to play R' is optimal for all possible beliefs p.

Given that Player 1 plays R, the value of p has to be equal 0, i.e. $p = 0$.

Given the player 2's strategy, for player 1 is optimal to play R

Then the strategy profile $\{(R),(R')\}$ with $p = 0$ represents a PBE.

Example 6



Find all Nash equilibria and check they are PBE

Solution

Nash equilibria

		Player 2	
		L'	R'
Player 1	L	2, 0	1, 1
	R	4, 1	2, 0
	A	3, 5	3, 5

Two Nash equilibria:

a) $\{(A),(R')\}$

b) $\{(R),(L')\}$

Consider $\{(A),(R')\}$ and check if it is PBE.

For Player 2 to play R' is optimal for all beliefs $p \geq 0.5$. Given that the player 2's information set is out of the equilibrium path, beliefs can be arbitrary. Given the player 2's strategy, for player 1 is optimal to play A. Then we have **a set** of PBE: strategy profile $\{(A),(R')\}$ with any $p \geq 0.5$.

Consider $\{(R),(L')\}$ and check if it is PBE.

For Player 2 to play L' is optimal for all beliefs $p \leq 0.5$. Given that the player 2's information set is on the equilibrium path, beliefs have to be consistent with player 1's strategy. Applying Bayes rule we find that (when the information set of player 2 is reached) the probability that player 1 played L is $\frac{0}{0+1} = 0$. Then beliefs have to be equal 0, i.e. $p = 0$. And for $p = 0$ to play L' is optimal. Now we check player 1. Given the strategy of player 2, for player 1 is optimal to play R.

Then the strategy profile $\{(R),(L')\}$ with $p = 0$ is a PBE.

Consider $\{(R),(R')\}$ and check if it is PBE. For Player 2 to play R' is optimal for all possible beliefs p.

a) Given that Player 1 plays R, the value of p has to be equal 0, i.e. $p = 0$.
b) Given the player 2's strategy, for player 1 is optimal to play R
c) Then the strategy profile $\{(R),(R')\}$ with $p = 0$ represents a PBE.