UNIVERSITÀ DEGLI STUDI DI TRIESTE
DIPARTIMENTO INGEGNERIA E ARCHITETTURA
Prof. Ing. Gianni Pedrizzetti, PhD
Corso di BIOFLUIDODINAMICA - Laurea Magistrale in Ingegneria Clinica.

PROGRAMMA DEL CORSO (9CFU)

A. ELEMENTI INTRODUTTIVI

1. Concetti di base:
Definizioni generali, natura corpuscolare, schema di continuo e leggi che governano la meccanica. Materiali solidi, elasticità. Materiali fluidi, viscosità. Cenni di anatomia umana e fisiologia del sistema cardiovascolare. Microcircolazione e circolazione nei gradi vasi: trasporto e diffusione. Altri tipi di moto fluido di rilevanza clinica. Analisi dimensionale, numeri puri.

2. Statica dei fluidi
Equazioni integrali e differenziali della statica. Distribuzione della pressione, carico piezometrico. Misura della pressione. Forze su superfici piane. Esempi. Forze su superfici curve. Esempi.

3. Cinematica
Richiami: operatore Nabla, teoremi di Gauss e di Stokes. Descrizione Lagrangiana e Euleriana. Suddivisione del moto fluido in traslazione, rotazione e deformazione, linee di corrente e traiettorie. Teorema del trasporto.

B. LEGGI DI CONSERVAZIONE

4. Conservazione della massa
Equazioni integrali di conservazione della massa. Equazione di continuità per le correnti. Equazione di continuità in forma differenziale e suo significato.

5. Conservazione della quantità di moto
Equazioni integrali di conservazione della quantità di moto. Equazioni di conservazione della quantità di moto per le correnti. Equazioni differenziali di conservazione della quantità di moto. Equazione di Cauchy. Equazioni costitutive per il sangue, modello Newtoniano. Equazioni di Navier-Stokes, condizioni al contorno e fluidi ideali. Definizione di correnti rettilinee.

C. METODI ELEMENTARI DI ANALISI DEL MOTO

6. Bilancio energetico in regioni limitate (Bernoulli)
Teorema di Bernoulli per i fluidi ideali, il concetto del tubo di Pitot e le misure emodinamiche. Utilizzo del bilancio di Bernoulli per il calcolo delle perdite di pressione valvolari. Perdite di energia localizzate ed estensione del teorema di Bernoulli in presenza di fenomeni dissipativi. Correzione del teorema di Bernoulli per correnti.

7. Moto unidirezionale in condotti rettilinei
Concetto di strato limite; moto che incontra una superficie piana e moto di una superficie piana. Sviluppo dello strato limite all'ingresso di un condotto. Moto piano uniforme e stazionario: scorrimento, tra due pareti Moto in un vaso circolare, profilo di Poiseuille. Moto pulsante in un vaso circolare; numeri di Reynolds, Womersley, Strouhal; differenze del moto tra vasi piccoli e vasi grandi.

8. Moto quasi unidirezionale nei grandi vasi
Cenni preliminari di turbolenza. Equazioni di Reynolds, moto mediamente uniforme. Bilancio di massa nei vasi con riduzione di sezione e ramificazioni. Moto in condotti curvi. Deformazione di condotti elastici. Moto in condotti elastici, celerità. Propagazione impulsi nella rete vascolare. Moto in condotti collassabili.

D. METODI AVANZATI DI ANALISI DEL MOTO

9. Vorticità e separazione dello strato limite

Concetto di vorticità. Decomposizione di Stokes. Strato limite come strato vorticoso. Equazione della vorticità. Separazione dello strato limite e formazione di vortici. Formazione di anelli vorticosi. Interazione tra vortici, interazione tra vortici e pareti. Instabilità del moto laminare e sviluppo di turbolenza.

10. Moto separato nei grandi vasi

Concetto del WSS. Fluidodinamica e nascita e sviluppo di arteriosclerosi. Flussi cardiovascolari con separazione dello strato limite: variazioni di sezione, biforcazioni, ramificazioni e valvole. Stenosi nelle carotidi, nelle coronarie. Conseguenze, soluzioni terapeutiche, soluzioni chirurgiche. Fluidodinamica e aneurismi. Aneurismi aortici ed addominali: nascita, sviluppo. Conseguenze, soluzioni terapeutiche, soluzioni chirurgiche

11. Meccanica Cardiaca I

Anatomia del cuore, camere, valvole e l'importanza del ventricolo sinistro. Il ciclo cardiaco elettromeccanico. Fasi dinamiche di deformazione, flussi volumetrici, pressioni. Moto fluido all'interno del cuore. Formazione di vortici. Scomposizione del moto fluido in termini di transito. Forze dinamiche intraventricolari. Fluidodinamica di patologie cardiache: ischemia, scompenso.

12. Meccanica Cardiaca II

Valvola aortica: condizioni naturali e patologiche. Stenosi, insufficienza e rigurgito; conseguenze, soluzioni terapeutiche, soluzioni chirurgiche, valvole artificiali. Flusso transmitrale: condizioni naturali e patologiche. Stenosi, insufficienza e rigurgito; conseguenze, soluzioni terapeutiche, soluzioni chirurgiche, valvole artificiali. Malformazioni cardiache. Tetralogia di Fallot.

1 CONCETTI di BASE

S1

Mechanics of Biological Fluids = BioFluidMechanics

⇓

laws of motion or equilibrium

(Greeks: force to have motion)
Newton (17th century): Force to change motion / to have acceleration

but Newton only for particles of mass "m"

Aria, acqua = fluids (modello × molecole ≤ nm)
continuum

Sangue = RBC (50%) + plasma (50%)

S2-S5

RBC ↑ 8 × 2 μm

plasma ↖ acqua

$V_{RBC} \simeq \pi \cdot 4^2 \times 2 \simeq 100\ \mu m = 10^{-7}\ mm^3$

$N_{RBC} = 50\% \times 10^7 \simeq 5 \times 10^6$ per mm^3

modello corpuscolare o continuo

fluido continuo modello × large regions but fine effects remain!

Schema di continuo (LARGE SCALE, LARGER THAN INDIVID. ELEMENTS)

GLOBAL PROPS: M, V

LOCAL PROPS: $\rho = \lim_{V \to 0} \frac{M}{V} = \frac{dM}{dV}$ at a "point"

~~zero~~ still >> particles

$\Rightarrow M = \int_V \rho\, dV \ldots$

velocity, pressure, temperature ...

Governing physical laws (Mechanics)

MECHANICS

1. CONS. MASSA
2. " QdM (Newton law)

2b. " Momento QdM ...

PROPERTIES CHARAT. THE CONTINUUM

3. Legge di stato $V = f(p, T)$ or $\rho = f(p, T)$

IDEAL GAS $PV = nRT$

example

We USE

Hp 1 $T \approx$ constant $\Rightarrow \rho = f(p)$ not for environment Atm/ocean...

HP 2 $\downarrow \Delta p$ $\updownarrow \Delta V$

Mod. compr. cubica $\epsilon = -\dfrac{\Delta p}{\Delta V / V} \overset{V = \frac{M}{\rho}}{=} +\rho \dfrac{dp}{d\rho}$ ~~fluid~~ fluido $\rightarrow \infty$ incomprimibile

legge di stato $\Rightarrow \rho =$ costante

No termodinamica, no energia T solo energie meccanica kin + pot

Rete cardiovascolare Solidi Vs Fluidi

$\boxed{S6}$

Due "modelli" per descrivere fenomeni reali

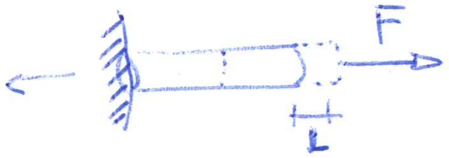
usage of one or another depends on the motion not just on the material

phenomenon scales

(example: ghiacciaio, sferette

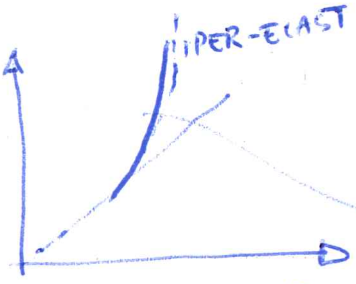
SOLIDS

[S7-S8]



$F \Rightarrow$ spostamento

TENS $\tau = F/A$ f.t. $\frac{\Delta L}{L} = \epsilon$ DEFORM



τ

ϵ

$\tau(\epsilon)$

ELAST. $\tau = E\epsilon$

IPERELAST. $\tau = c(e^{\alpha\epsilon} - 1)$

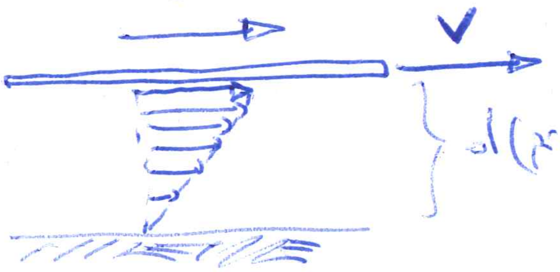
(REVERSIBILE)

PARAM. ELASTICITÀ

FLUIDS

[S9]

$\tau = F/A$



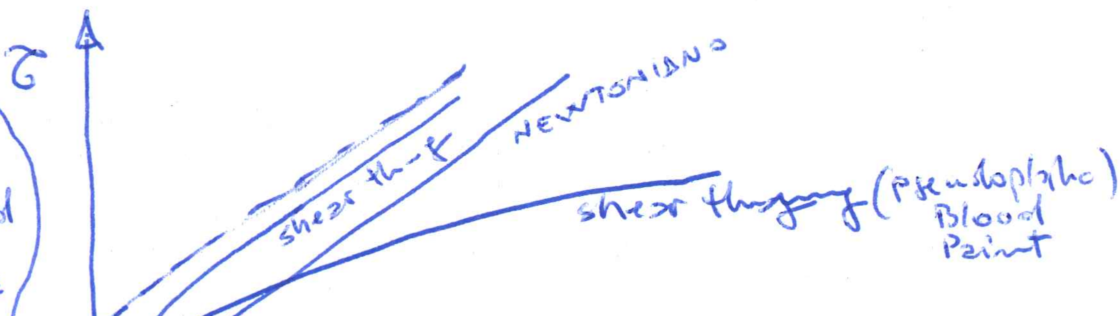
$$\tau = f\left(\frac{V}{d}\right) = f\left(\frac{\partial V_x}{\partial y}\right)$$

$\mu = \begin{cases} 10^{-3} \text{ Acqua} \\ 1{,}8 \times 10^{-5} \text{ Aria} \\ 3.3 \times 10^{-3} \text{ Blood} \end{cases}$

$10^{-3}\ Pa \cdot s = \text{cPoise}$

$\nu = \frac{\mu}{\rho} \begin{cases} 10^{-6}\ m^2/s \text{ Acqua} \\ 1.5 \times 10^{-5}\ \frac{m^2}{s} \text{ Air} \\ 3{,}3 \times 10^{-6} \text{ BLOOD} \end{cases}$

$10^{-6}\ \frac{m^2}{s} = 1\ \frac{mm^2}{s}$



NEWTONIAN

$$\tau = \mu \frac{\partial V_{r\theta}}{\partial n}$$

$\mu\left(\frac{\partial V_{r\theta}}{\partial n}\right)$ decreasing with higher shear rate

sh. thinning (BLOOD)

PARAM. VISCOSITÀ

Biofluids:

- pulmonary circulation
  FLUID = AIR   forced in/out motion to reach alveoli
  main phs   alveoli dispiegati/appiattiti collapse for high pressure

- Eye   Column flow in/out, vitreo several phases, mostly a balance

- Respiration Airways . . . . turbolenti nasali muco bronchiale . . . .

- perfusion phenomena

  ⋮

- clinical labs
- . . . plants

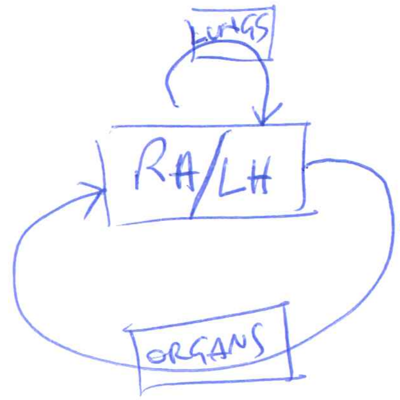
other engineers but we have to understand

  ⋮

- Blood circulation

S1d

Sistema circolatorio



2 circ. in parallel
2 "indep." pumps

ARTERIAL SYSTEM ; unsteady flow, high pressure (thus hidden) 80-120 mmHg

VENOUS SYST. ~ steady flow, low press. ≃ Atm. 1-1,5 m_{H_2O}

Some Anatomy info (see S6)

FUNCTION REACH ALL BODY CELLS!

COVER High Distances ~ 1 m

→ TRANSPORT $\quad l_{TRAS} \simeq Vel. \times t$ $\quad$ Efficient × LARGE SCALE (ALL BODY) $\quad V \sim cm/s$

not efficient for small scale, vessels $V \to 0$ otherwise huge shear

→ DIFFUSION $\quad \dfrac{\partial f}{\partial t} = D \dfrac{\partial^2 f}{\partial x^2}$

Solution (1D) $\quad f(x,t) = \dfrac{1}{\sigma(t)} e^{-\frac{x^2}{2\sigma^2}}$ $\quad$ S11-S12

$$l_{DIFF} \simeq \sigma = \sqrt{2Dt}$$

EFFicient × SMALL SCALES (ALL CELLS)

t	l_{TRAS} $V = 20\ cm/s$	l_{TRASP} $V = 2\ mm/s$	l_{DIFF} $D \simeq 3.5 \times 10^{-5}\ m^2/s$
1 min	12 m	12 cm	1,5 cm
1 s	20 cm	$\boxed{2\ mm}$	$\boxed{\sim 2\ mm}$
0,1 s	2 cm	0,2 mm	0,6 mm
10^{-2} s	2 mm	20 μm	0,2 mm = 200 μm
10^{-3} s	0,2 mm	2 μm	60 μm

MULTI-SCALE SYSTEM

45 B

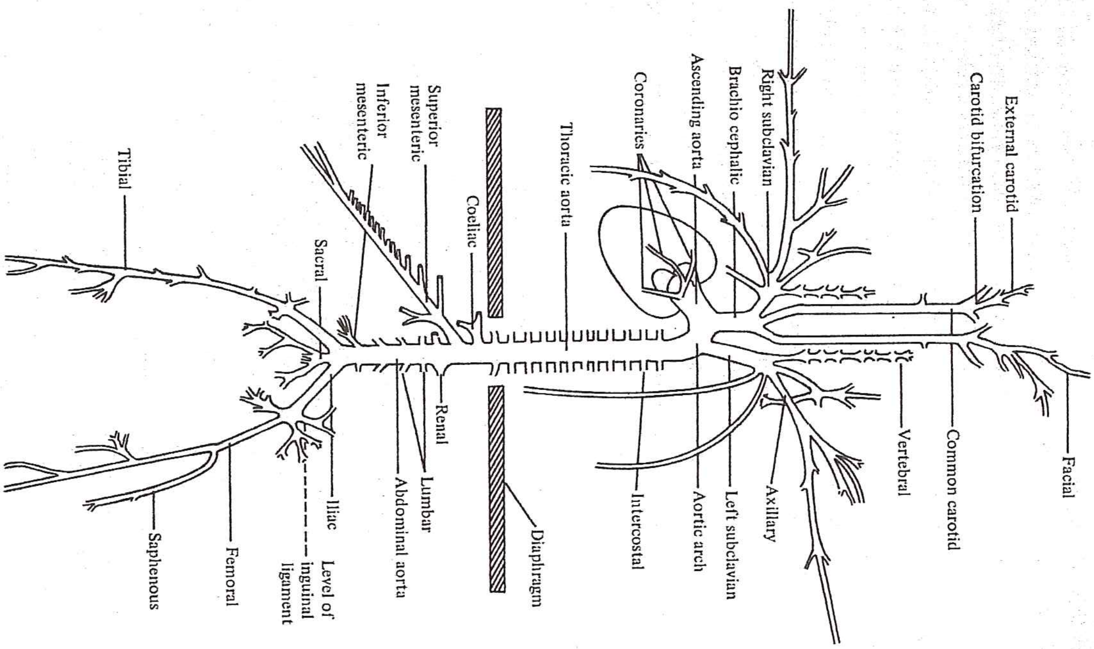


Fig. 1.3. A diagrammatic representation of the major branches of the canine arterial tree. (After McDonald, 1974.)

Table 1.1. *Normal values for canine cardiovascular parameters. An approximate average value, and then the range, is given where possible. All values are for the dog except those for arteriole, capillary, and venule, which have only been measured in smaller mammals*

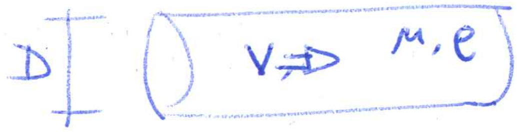
Site	Ascending aorta	Descending aorta	Abdominal aorta	Femoral artery	Carotid artery	Arteriole	Capillary	Venule	Inferior vena cava	Main pulmonary artery
Internal diameter, d_i (cm)	1.5 1.0–2.4	1.3 0.8–1.8	0.9 0.5–1.2	0.4 0.2–0.8	0.5 0.2–0.8	0.005 0.001–0.008	0.0006 0.0004–0.0008	0.004 0.001–0.0075	1.0 0.6–1.5	1.7 1.0–2.0
Wall thickness, h (cm)	0.065 0.05–0.08		0.05 0.04–0.06	0.04 0.02–0.06	0.03 0.02–0.04	0.002	0.0001	0.0002	0.015 0.01–0.02	0.02 0.01–0.03
h/d_i	0.07 0.055–0.084		0.06 0.04–0.09	0.07 0.055–0.11	0.08 0.053–0.095	0.4	0.17	0.05	0.015	0.01
In-vivo length (cm)	5	20	15	10	15 10–20	0.15 0.1–0.2	0.06 0.02–0.1	0.15 0.1–0.2	30 20–40	3.5 3–4
Approximate cross-sectional area (cm^2)	2	1.3	0.6	0.2	0.2	2×10^{-5}	3×10^{-7}	2×10^{-5}	0.8	2.3
Total vascular cross-sectional area at each level (cm^2)	2	2	2	3	3	125	600	570	3.0	2.3
Peak blood velocity ($m\,s^{-1}$)	1.2 0.4–2.9	1.05 0.25–2.5	0.55 0.5–0.6	1.0 1.0–1.2		0.75	0.07	0.35	0.25 0.15–0.4	0.7
Mean blood velocity ($m\,s^{-1}$)	0.2 0.1–0.4	0.2 0.1–0.4	0.15 0.08–0.2	0.1 0.1–0.15		0.005–0.01	0.0002–0.0017	0.002–0.005		0.15 0.06–0.28
Peak Reynolds number, $\hat{Re}$	4500	3400	1250	1000		0.09	0.001	0.035	700	3000
Frequency parameter, α (heart-rate 2 Hz)	13.2	11.5	8	3.5	4.4	0.04	0.005	0.035	8.8	15
Calculated wave speed, c_0 ($m\,s^{-1}$)	5.8		7.7	8.4	8.5				1.0	3.5
Measured wave speed, c ($m\,s^{-1}$)	5.0 4.0–6.0		7.0 6.0–7.5	9.0 8.0–10.3	8.0 6.0–11.0				4.0 1.0–7.0	2.5 2.0–3.3
Young's modulus, $E(\times10^2\,kN\,m^{-2})$	4.8 3–7		10 9–11	10 9–12	9 7–11				0.7 0.4–1.0	6 2–10

Analisi dimensionale

espr. grandezza fisica = $\overset{\text{numero}}{A} \times UNIT_1 = \overset{\text{numero}}{B} \times UNIT_2$

es. altezza uomo $1,80\,m = 180\,cm = 70,87\,inch$
INDIP. DA UNITS.

Espr. leggi fisica anche INDIP. DA UNITS.
$\Rightarrow$ semplific. !!



Perdite x attrito $\underset{N/m^3}{\dfrac{\Delta P}{L}} = f(V, D, \underset{Pa\cdot s}{\mu}, \rho)$

$[L] = D \;;\; [T] = \dfrac{D}{V} \;;\; [M] = \rho D^3$

$\left[\dfrac{\Delta P}{L}\right] = \rho\dfrac{V^2}{D} \;;\; [\mu] = \rho V D$

$$\frac{\Delta P}{L} = \rho\frac{V^2}{D}\, f\left(1, 1, \frac{\mu}{\rho V D}, 1\right) = \rho\frac{V^2}{D}\, f(Re)$$

in generale $q = f(a_1, a_2 .. a_N) \Rightarrow \tilde{q} = f(\tilde{a}_1 \tilde{a}_2 \ldots a_{N-D})$
SEMPRE LEGGI IN FORMA ADIMENSIONALE

$$\frac{\Delta P}{L \,/\, \frac{\rho V^2}{D}} = f(V, D, \mu, \rho, T) = f(Re, St)$$

or $[T] = T$

.... stesso risultato

$Re = \dfrac{VD}{\nu} \;;\; St = \dfrac{D}{VT} \;;\; Wo = \dfrac{D}{\sqrt{\nu T}} \quad \left(St = \dfrac{D^2}{\nu T}\right)$

SOLO PAR. ADIMENS. × GENERAL UNIVERSAL

2 ~~IDRO~~STATICA DEI FLUIDI

$$v \equiv 0 \qquad \tau_{rg} \equiv 0 \Rightarrow \vec{\tau} = p\,\vec{n}$$

SCALAR

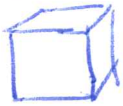
Eq. Integrali

$$\int \bar{f}\, dV + \int \overset{p\bar{n}}{\bar{\tau}}\, dA = 0$$

GRAVIT. $\bar{f} = -\nabla \gamma z$ altrimenti others

$$\int p\,\bar{n}\, dA = \gamma V \vec{k}$$

Eq. Differenziali

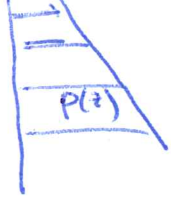
p  $p + \frac{\partial p}{\partial x} dx$

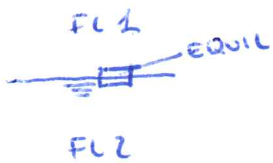
$$\begin{cases} \frac{\partial p}{\partial x} = f_x \\ \frac{\partial p}{\partial y} = f_y \\ \frac{\partial p}{\partial z} = f_z \end{cases} \Rightarrow \nabla p = \vec{f}$$

GRAVIT. $\bar{f} = -\nabla \gamma z$

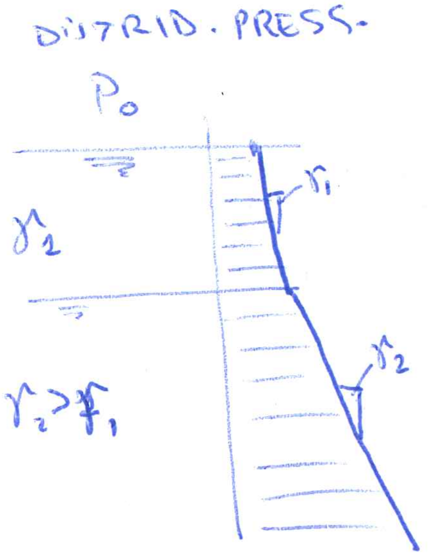
$$\Longrightarrow \nabla(p + \gamma z) = 0 \qquad \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0 \Rightarrow p(z), \quad \frac{\partial p}{\partial z} = -\gamma$$

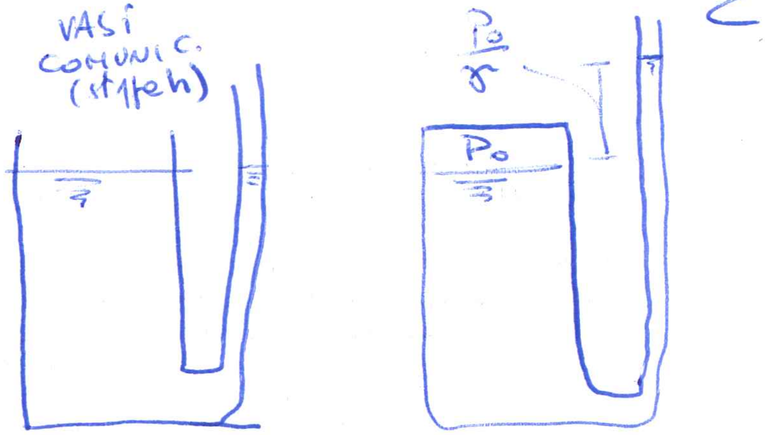
(*) $h \triangleq z + \frac{p}{\gamma}$ è costante in uno stesso fluido



(*) 

p è continua all'interfaccia tra 2 fluidi (ignora tens. sup./capill.)





Misura della pressione

$$\frac{F}{A} \Rightarrow \frac{N}{m^2} = Pa \qquad \text{altezza di fluido } h_{fl.} = \frac{P}{\gamma_{fl.}}$$

$$P\,[Pa] = \overbrace{9810 \frac{N}{m^3}}^{\gamma_{H_2O}} \times h_{H_2O}[m]$$

$$= \underbrace{133 \times 10^3 \frac{N}{m^3}}_{\gamma_{Hg}} \times \underbrace{h_{Hg}[mm] \times 10^{-3}}_{h\,[m]} = 133 \times h_{mmHg}$$

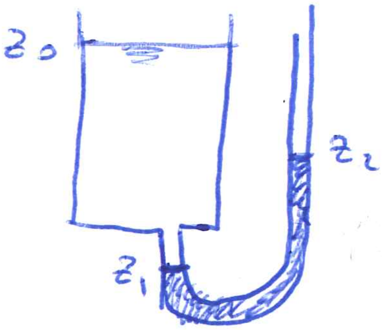
$$P\,[kPa] = 7{,}5\ P\,[mm_{Hg}]$$

$$P\,[mm_{Hg}] = 133\ P\,[Pa]$$

$$1\,[Atm] = 101\ kPa = 1{,}01\ bar$$

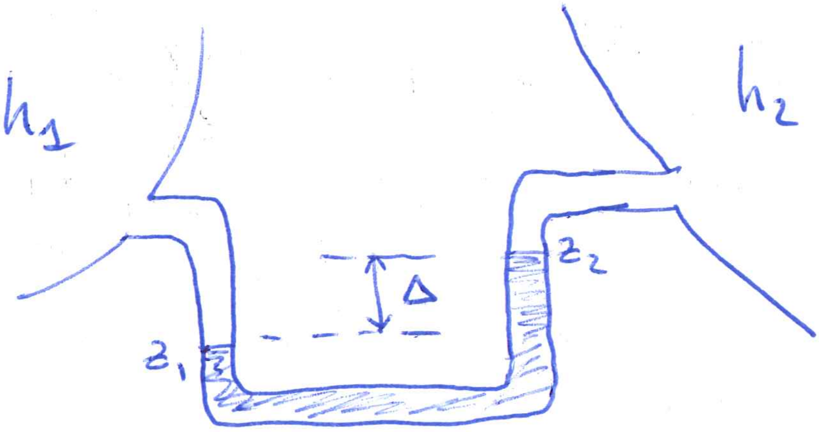
$$= 10{,}3\ m_{H_2O} = 76\ mm_{Hg}$$

CONCETTO BASE MANOMETRO



$$\left.\begin{aligned} z_0 + \frac{\cancel{p_0}}{\gamma} &= z_1 + \frac{p_1}{\gamma} \\ z_1 + \frac{p_1}{\gamma_m} &= z_2 + \frac{\cancel{p_0}}{\gamma_m} \end{aligned}\right\} \gamma h_{H_2O} = \gamma_m h_m$$

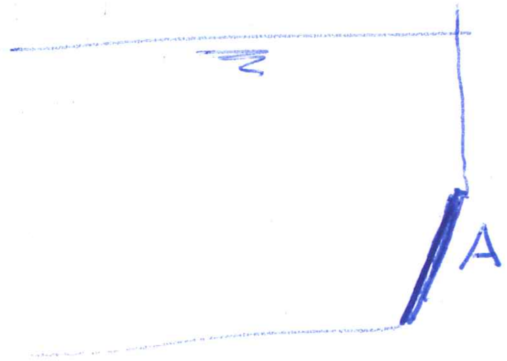
MANOMETRO DIFFERENZIALE



DEF $\quad h_1 - h_2 = z_1 + \frac{p_1}{\gamma} - z_2 - \frac{p_2}{\gamma} = \frac{p_1 - p_2}{\gamma} - \Delta = \left(\frac{\gamma_m}{\gamma} - 1\right)\Delta$

$z_1 + \frac{p_1}{\gamma_m} = z_2 + \frac{p_1}{\gamma_m} \qquad p_1 - p_2 = \gamma_m \Delta$

FORZE SU SUP. PIANE



$$\vec{F} = \int p\, \vec{n}\, dA = F\, \vec{n}$$

$$p(z) = p_0 + \gamma (z_0 - z)$$

$$F = \int p\, dA = p_0 A + \gamma z_0 A + \gamma \underbrace{\int_A z\, dA}_{z_G A} = \left[p_0 + \gamma(z_0 - z_G)\right] A = p_G A$$

press. baric. × A

$$\boxed{F = p(z_G)\, A}$$

risultato generale xché lineare

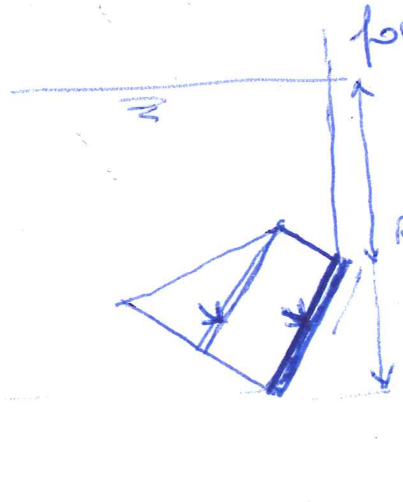
N.B. NON è applicata in G

$$M = \int p(z)\, b(x,y,z)\, dA$$

no gen. formula but usually easy

centro d'applicazione è baricentro volume della pressione

for example



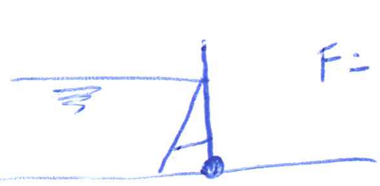
RETTANG.

$$F = \gamma \frac{(h_1 + h_2)}{2} (h_2 - h_1) B \quad \text{AREA TRAPEZ.}$$

$$F_1 = \gamma h_1 (h_2 - h_1) B \qquad b_1 = \frac{1}{2} L_G$$

$$F_2 = \gamma (h_2 - h_1)(h_2 - h_1) \qquad b_2 = \frac{2}{3}$$

RETTANG. AFFIORANTE



$$F = \gamma \frac{h^2}{2} \qquad b = \frac{h}{3}$$

ESEMPI

FORZE SU SUP. CURVE

CORPO CHIUSO

Archimede III a.C.

$F_{orizz.} = 0$

sostit. corpo con acqua

$G_z + \Pi_z = 0$

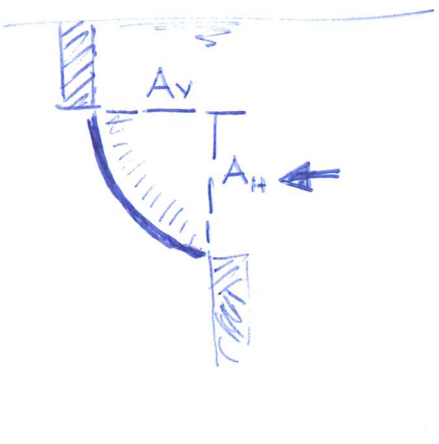
$-\gamma V + \underbrace{\int p \underline{n} \, dA}_{F_z} = 0$ $\boxed{F_z = \gamma V}$

se non è acqua ma stell. equil. → distrib. $p(z)$ stessa

$F_z = \gamma V$ "un corpo immerso riceve una spinta verso l'alto pari al peso...."

SUP. APERTA

$\underline{G} + \underline{\Pi} = 0$ Volume di controllo

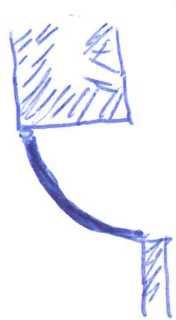


orizz.) $\Pi_{A_H} + F_H = 0$ $F_H = p_{G_H} A_H$

vert.) $-\gamma V_c + F_V + \Pi_{A_V} = 0$

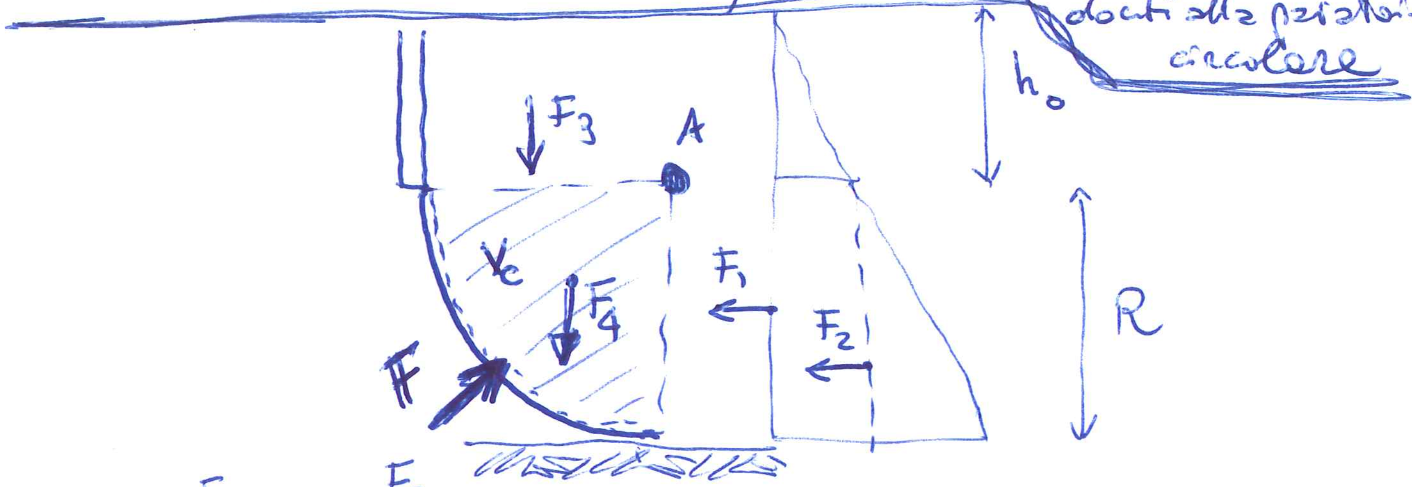
$F_V = \gamma V_c + p_{G_V} A_V = \gamma V_{TOT}$

1. si possono prendere vol. diversi (∀, comodo, arbitrario) orizz. vertic. G+Π vale sempre
2. idealizz. pb. per non cascare in tranelli



3. Momenti bilancio MθdM

Si calcoli F e M agenti sull'albero di rotaz. A dovuti alla paratoia circolare



$$F_h = \overset{F_1}{\gamma h_0 R} + \overset{F_2}{\gamma \frac{R^2}{2}}$$

$$F_v = \underset{F_3}{\gamma h_0 R} + \underset{F_4}{\gamma \pi \frac{R^2}{4}}$$

$$M = \cancel{\gamma h_0 R \cdot \underset{R/2}{b_1}} + \gamma \frac{R^2}{2} \underset{\frac{2}{3}R}{b_2} - \cancel{\gamma h_0 R \underset{R/2}{b_3}} - \gamma \pi \frac{R^2}{4} b_4$$

b_4: baricentro x di $\frac{1}{4}$ cyl. $\frac{4}{3\pi}R$

$$b_4 \triangleq \frac{1}{A}\int_0^R \int_0^{\pi/2} x \, dA = \frac{1}{A}\iint r\cos\vartheta \, r \, d\vartheta \, dr = \frac{1}{A}\int_0^R \left\{ r^2 \underbrace{\int_0^{\pi/2} \cos\vartheta \, d\vartheta}_{=1} \right\} dr =$$

$$= \frac{1}{A}\int_0^R r^2 dr = \frac{R^3}{3}\frac{4}{\pi R^2} = \frac{4}{3\pi}R$$

$$M \equiv 0 \;!!$$

infatti $F = \int \overbrace{p \bar{n} \, dS}^{\text{forzelline dirette verso A, braccio nullo}}$

ESEMPI SPINTE IDROST

3 KYNEMATICS NABLA

VECTOR OPERATOR $\nabla = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$

GRADIENT (INCREASE)

scalar $\nabla\phi = \begin{bmatrix} \\ \\ \end{bmatrix} = \frac{\partial\phi}{\partial x_i}$

a vector $\perp$ to $\phi = \text{const}$

$$\frac{\partial\phi}{\partial n} = \nabla\phi \cdot n$$

~~DIVERGENCE~~

vector $\nabla \underline{v} = [\nabla v_x ; \nabla v_y ; \nabla v_z] = \begin{bmatrix} \\ \end{bmatrix} = \frac{\partial v_i}{\partial x_j}$

DIVERGENCE (DECREASE)

no scalar

vector $\nabla \cdot \underline{v} = \frac{\partial v_i}{\partial x_i} \rightarrow$ $\nabla \cdot v = 0$ SOLENOIDAL

why the name

tensor $\nabla \cdot \underline{\underline{T}} = \frac{\partial T_{ij}}{\partial x_j} \ldots \begin{bmatrix} - \\ - \\ - \end{bmatrix}$

CURL (equal)

$$\nabla \times \underline{v} = \begin{bmatrix} \frac{\partial v_3}{\partial x_2} - \frac{\partial v_2}{\partial x_3} \\ - \; - \\ - \; - \end{bmatrix}$$

$\nabla \times v = 0$ IRROTATIONAL

if v = vel. $\nabla \times v$ (velocity) is 2x angul. vel.

$\nabla \cdot \nabla \times v \equiv 0$; $\nabla \times \nabla v \equiv 0$

GAUSS THEOREM (o della divergenza)

~~[illegible]~~ V (normale uscente)

$$\int_V \nabla \cdot \underline{v} \, dV = \int_{A=\partial V} \underline{v} \cdot \underline{n} \, dA$$

divergenza — flusso

STOKES THEOREM (o della circolazione)

$$\int_A (\nabla \times v) \cdot n \, dA = \int_{C=\partial A} v \cdot ds$$

vel. angolare $\perp A$ — rotazione

$$2\omega \pi R^2 = u_\vartheta 2\pi R \Rightarrow v_\vartheta = \omega R$$

CINEMATICA

Scomposizione atto di moto

TAYLOR

$$\bar{v}(\bar{x}+d\bar{x}) = \bar{v}(x) + \nabla v \cdot d\bar{x}$$

$$v_i(\bar{x}+dx) = \underbrace{v_i(x)}_{\text{TRASL. ok}} + \underbrace{\frac{\partial v_i}{\partial x_j}} dx_j$$

$$\frac{\partial v_i}{\partial x_j} = \underbrace{\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)}_{\text{SYM}} + \underbrace{\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right)}_{\text{ANTISYM}} \quad ; \quad \nabla v = \mathbb{D} + \Omega$$

$$v(x+dx) = v(x) + \mathbb{D}\cdot dx + \cancel{\Omega \cdot dx}$$

$$\frac{1}{2}\bar{\omega} \times d\bar{x}$$

$$\nabla \times \bar{v}$$

TRASLAZ. RIGIDA

ROTAZ. RIGIDA

DEFORM

DEVIAT

isotr.

$$\mathbb{D} = tr(\mathbb{D})\,\mathbb{I} + \left(\mathbb{D} - tr(\mathbb{D})\,\mathbb{I}\right)$$

ESPANS. PURA

DEF. PURA

RESP. ATTR. INTERNI

τ

$$v_i = (\nabla \cdot v)\, dx_i$$

Zero se incompress.

Linee di corrente

Traiettorie

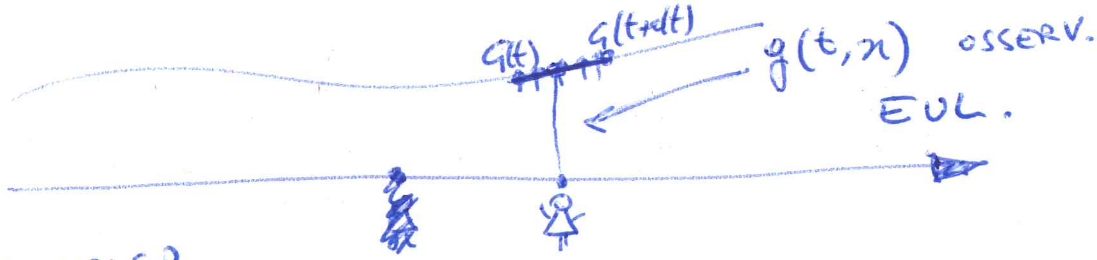
CINEMATICA

Leggi fisiche applicate elem. materiali:

elemento identif. posiz. $X(t)$ } LAGR.

proprietà identif. da grandezza $G(t)$

1D



CORRISP.

$$g(t, X(t)) = G(t) \;\dots\dots$$

$$\underbrace{\frac{dG}{dt}}_{\text{deriv. LAGR.}} = \frac{\partial g}{\partial t} + \frac{\partial g}{\partial x}\frac{\partial X}{\partial t} = \underbrace{\frac{\partial g}{\partial t} + v\frac{\partial g}{\partial x}}_{\text{IN TERM. EULER.}}$$

3D

$$\frac{dG}{dt} = \frac{\partial g}{\partial t} + v_x\frac{\partial g}{\partial x} + v_y\frac{\partial g}{\partial y} + v_z\frac{\partial g}{\partial z} = \frac{\partial g}{\partial t} + \vec{v}\cdot\nabla g$$

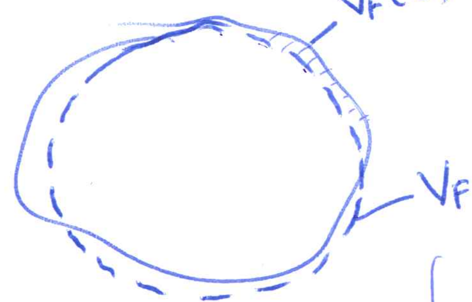
$\Rightarrow$ ACCELERAZ. OF A FLUID PARTICLE

$$a = \overbrace{\frac{d\bar{v}}{dt}}^{\text{LAGR}} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} \quad (\text{IN 3D}) \quad = \frac{\partial \bar{v}}{\partial t} + \bar{v}\cdot\nabla\bar{v}$$

$\boxed{S13 \div S15}$

Th. TRASPORTO

Trasf. leggi di conserv. globali da un volume fluido a un volume fisso
LAGR EUL



$$\frac{d}{dt}\int_{V_F(t)} G(t)\,dV = \frac{1}{dt}\left\{\int_{V_F(t+dt)} g(t+dt)\,dV - \int_{V(t)} g(t)\,dV\right\} =$$

$$= \frac{1}{dt}\left\{\int_{V(t)} g(t+dt)\,dV + \int_{V(t+dt)-V(t)} g(t+dt)\,dV - \int_{V(t)} g(t)\,dV\right\} =$$

ord sup. norm. USCENTE

$$= \int_{V(t)=V} \frac{\partial g}{\partial t}\,dV + \frac{1}{\cancel{dt}}\int_{S(t)} \left(g(t) + \cancel{\frac{\partial g}{\partial t}dt}\right)\left(+\bar{v}\cdot\bar{n}\,\cancel{dt}\,dS\right) =$$

$$= \int_V \frac{\partial g}{\partial t}\,dV + \int_S g\,\bar{v}\cdot\bar{n}\,dS = \int_V \left\{\frac{\partial g}{\partial t} + \bar{\nabla}\cdot(g\bar{v})\right\}dV$$

volumi fissi
EUL.

S16

1

4 CONS. MASSA

$$0 = \frac{d}{dt}\int_{V_F} \rho\, dV \overset{\text{th. TRASP}}{=} \int_V \frac{\partial \rho}{\partial t} dV + \int_S \rho \bar{v}\cdot\bar{n}\, dS$$

INCOMPR. $\Rightarrow$ $\int_S \bar{v}\cdot\bar{n}\, dS = 0$

$v_n \cdot S_{LAT} = \frac{dV}{dt}$

$$(Q_2 - Q_1) + \frac{dV}{dt} = 0$$

S17

Tronco di lungh. dx $\quad dV = A\cdot dx$

$$\boxed{\frac{dQ}{dx} + \frac{dA}{dt} = 0}$$

Eq. CONT. CORRENTE

explanation

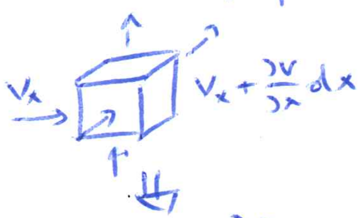
th. GAUSS. $\int_S (v\cdot n)\, dS = \int_V \nabla\cdot\bar{v}\, dV$

$\forall V \Rightarrow$ $\boxed{\nabla\cdot\bar{v} = 0}$

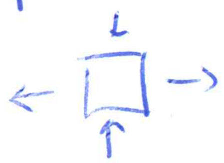
eq. cont. forma diff.

explanations

similarly franfran:o



$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$



S18

5 CONS. QdM (F=ma)

$$\frac{d}{dt}\int_{V_f(t)} \rho \bar{v}\, dV = \int_V \bar{f}\, dV + \int_S \bar{\sigma}\, dS$$

Th. Trasporto

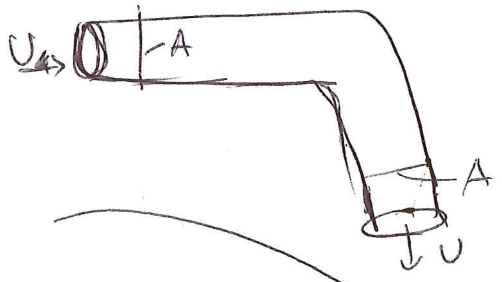
EQ. INTEGRALE

$$\int_V \frac{\partial}{\partial t}(\rho \bar{v})\, dV + \int_S \rho \bar{v}(\bar{v}\cdot\bar{n})\, dS = \int_V \bar{f}\, dV + \int_S \bar{\sigma}\, dS$$

n "uscente"

$$\mathbb{I} + \mathbb{M} = \mathbb{G} + \Pi$$

useful for hydrodyn forces



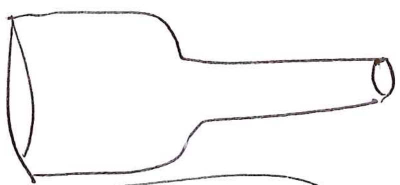
$M_x = \bar{V} U^2 A$, $M_y = \bar{V} U^2 A$

$\bar{\mathbb{I}} \equiv 0$ (STAZ.) $\bar{\mathbb{G}} \equiv 0$ SUL PIANO

$\Pi_x = p_1 A - F_x$

$\Pi_y = + p_2 A - F_y$

$$\bar{F} = \begin{bmatrix} p_1 A + U^2 A \\ + p_2 A + U^2 A \end{bmatrix} \begin{matrix} \rightarrow \\ \uparrow \end{matrix}$$

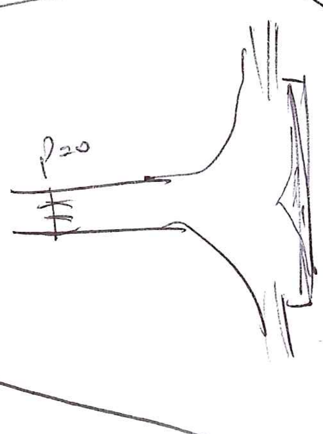


$I_x = \int \rho \frac{\partial v}{\partial t}\, dA\, dx = \rho A \frac{dU}{dt} L$ ACC $\rightarrow$ DEC $\leftarrow$

$M_x = -\rho \frac{Q^2}{A_1} + \rho \frac{Q^2}{A_2}$ $\quad G = 0$

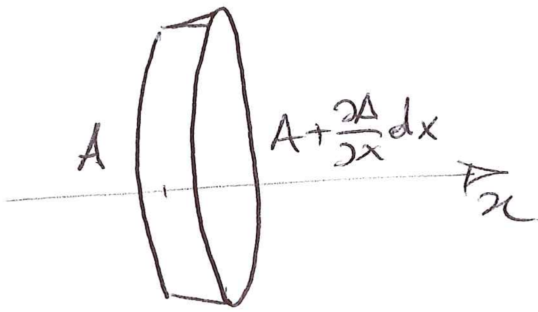
$\Pi_x = p_1 A_1 - p_2 A_2 - F_x$

$F_x = p_1 A - p_2 A_2 - \rho Q^2 \left(\frac{1}{A_1} - \frac{1}{A_2}\right)$ REAZ.



STAZ. $F_x = \rho U^2 A = \rho Q U$

CORRENTI



$$U = \frac{1}{A}\int_A v_x \, dA$$

$$\mathrm{I} = \int \rho \frac{\partial v}{\partial t} dV \Rightarrow \int \rho \frac{\partial v}{\partial t} dA\, dx = \rho \frac{\partial U}{\partial t} A\, dx$$

$$M = \int_S \rho\, v_x (\bar{v}\cdot\bar{n}) = -\int_{A(x)} \rho v^2 dA + \int_{A(x)+\frac{\partial A}{\partial x}dx} \rho \left(v+\frac{\partial v}{\partial x}dx\right)^2 dA + \int_C \rho\, v_x v_n \, dC\, dx =$$

$$\underset{\alpha\approx 1}{=} -\rho U^2 A + \int_{A(x)} \rho\left(v+\frac{\partial v}{\partial x}dx\right)^2 dA + \int_{\frac{\partial A}{\partial x}dx} \rho \left(v+\frac{\partial v}{\partial x}dx\right)^2 dA + \rho U \frac{\partial A}{\partial t} dx =$$

$$= \cancel{-\rho U^2 A} + \cancel{\rho U^2 A} + 2\rho U \frac{\partial U}{\partial x} A\, dx + O(dx^2) + \rho U^2 \frac{\partial A}{\partial x} dx + O(dx^2, dx^3) + \rho U \frac{\partial A}{\partial t} dx$$

$$= \rho A\, dx \cdot \left\{ U\frac{\partial U}{\partial x} + \underbrace{U\frac{\partial U}{\partial x} + \frac{U^2}{A}\frac{\partial A}{\partial x} + \frac{U}{A}\frac{\partial A}{\partial t}}_{\frac{U}{A}\times \text{CONT} \equiv 0} \right\} = \rho A U \frac{\partial U}{\partial x} dx$$

$$\Pi = pA - \left(p+\frac{\partial p}{\partial x}dx\right)\left(A+\frac{\partial A}{\partial x}dx\right) + \left(p+\frac{\partial p}{2\partial x}dx\right)\frac{\partial A}{\partial x}dx - \tau_w C\, dx =$$

$$= -\frac{\partial p}{\partial x} A\, dx - \tau_w C\, dx$$

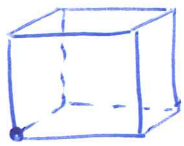
$$G = -\gamma \underbrace{A\, dx}_{V} \frac{d\hat{z}}{dx}$$

$$\boxed{\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -\frac{\partial}{\partial x}\overbrace{\left(\frac{p}{\rho} + g\hat{z}\right)}^{gh} - \frac{\tau_w}{\rho}\frac{C}{A}}$$

CONS. QdM   FORMA DIFFERENZIALE

$$\int \frac{\partial v}{\partial t}\, dV + \int \underset{\sim}{v}\,(\underset{\sim}{v}\cdot\underset{\sim}{n})\, dA = \frac{1}{\rho}\int \underset{\sim}{f}\, dV + \frac{1}{\rho}\int \underset{\sim}{\tau}\, dA$$

ASSE X



I $\quad \frac{\partial v_x}{\partial t}\, dx\,dy\,dz$

II $\quad - v_x v_x\, dy\,dz - v_x v_y\, dx\,dz - v_x v_z\, dx\,dy$

$$+ \left(v_x + \frac{\partial v_x}{\partial x}dx\right)\left(v_x + \frac{\partial v_x}{\partial x}dx\right)dy\,dz + \left(v_x + \frac{\partial v_x}{\partial y}dy\right)\left(v_y + \frac{\partial v_y}{\partial y}dy\right)dx\,dz + ()()$$

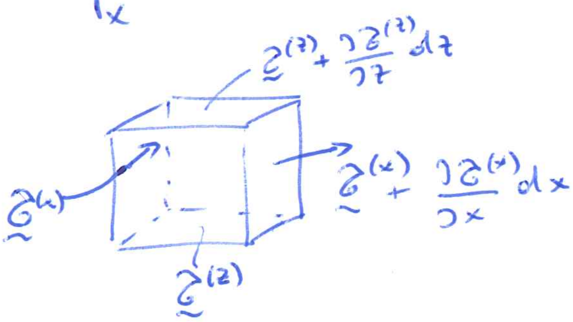
$$= \left(\cancel{2}\, v_x \frac{\partial v_x}{\partial x} + \cancel{v_x \frac{\partial v_y}{\partial y}} + v_y \frac{\partial v_x}{\partial y} + \cancel{v_x \frac{\partial v_z}{\partial z}} + v_z \frac{\partial v_x}{\partial z}\right) dx\,dy\,dz$$

$+$ termini dx^2

$$= \left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) dx\,dy\,dz$$

III $= f_x\, dx\,dy\,dz$ ...GRAVIT.... $\frac{\partial}{\partial x}\left(-\gamma \hat{z}\right) dx\,dy\,dz$

IV



— IDROST

$\underset{\sim}{\tau}^{(x)} = p\, \underset{\sim}{n}^{(x)} = \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix}$

$\underset{\sim}{\tau}^{(y)} = p\, \underset{\sim}{n}^{(y)} = \begin{bmatrix} 0 \\ p \\ 0 \end{bmatrix}$

...

$$- \tau_x^{(x)}\, dy\,dz + \left(\tau_x^{(x)} + \frac{\partial \tau_x^{(x)}}{\partial x} dx\right) dy\,dz + \dots$$

$$= + \left(\frac{\partial \tau_x^{(x)}}{\partial x} + \frac{\partial \tau_x^{(y)}}{\partial y} + \frac{\partial \tau_x^{(z)}}{\partial z}\right) dx\,dy\,dz$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = \frac{1}{\rho} f_x + \frac{1}{\rho} \left\{ \frac{\partial \tau_x^{(x)}}{\partial x} + \frac{\partial \tau_x^{(y)}}{\partial y} + \frac{\partial \tau_x^{(z)}}{\partial z} \right\}$$

$$\text{DEF} \quad \mathbb{T}_{ij} = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ - & - & - \\ - & - & - \end{bmatrix}_{ij} = \tau_i^{(j)}$$

"TENSORE" DELLE TENSIONI

$$\frac{\partial \bar{v}}{\partial t} + \bar{v} \cdot \nabla \bar{v} = \frac{1}{\rho} \bar{f} + \frac{1}{\rho} \nabla \cdot \mathbb{T}$$

EQ. CAUCHY

gen. x ogni continuo

I) potevo arrivarci in generale?

$$\int_V \frac{\partial v_x}{\partial t} dV + \int_S \bar{v}(\bar{v}\cdot\bar{n})\, dS = \frac{1}{\rho}\int_V \bar{f}\, dV + \frac{1}{\rho}\int_S \tau\, dA$$

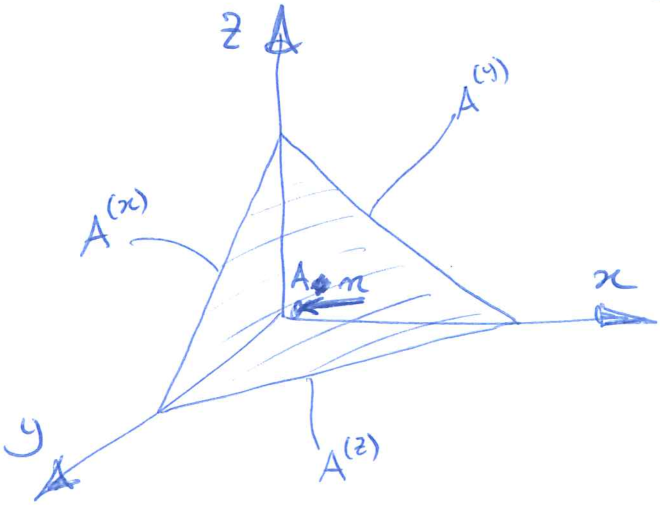
$$\int_S v_i v_j n_j\, dS \overset{GAUSS}{=} \int_V \frac{\partial}{\partial x_j} v_i v_j\, dV = \int \cancel{v_i \frac{\partial v_j}{\partial x}}^{=0} dV + \int_V v_j \frac{\partial v_i}{\partial x_j} dV$$

$$\frac{1}{\rho}\int_V \nabla \cdot \mathbb{T}\, dV \overset{\substack{REV\\GAUSS}}{=} \int_S \mathbb{T} \cdot n\, dS$$

$$\Rightarrow \bar{\tau} = \bar{\bar{\mathbb{T}}} \cdot \bar{n}$$

TETRAEDRO CAUCHY

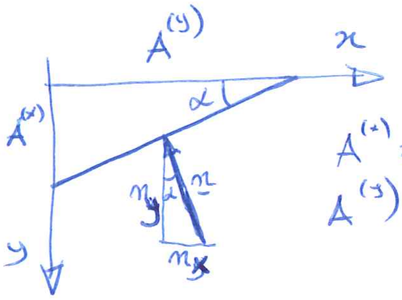
TETRAEDRO DI CAUCHY



$$\underset{\sim}{\tau} A_0 + \left(\underset{\sim}{\tau}^{(x)} A^{(x)} + \underset{\sim}{\tau}^{(y)} A^{(y)} + \underset{\sim}{\tau}^{(z)} A^{(z)} \right) = 0$$

ma $A^{(x)} = -A_0 n_x$ ...

anche per $A_0 \to 0$ (per cui punto è in equilibrio interno)



$A^{(x)} = A_0 \sin\alpha$

$A^{(y)} = A_0 \cos\alpha$

$n_x = -\sin\alpha$

$n_y = -\cos\alpha$

$$\underset{\sim}{\tau} = \underset{\sim}{\tau}^{(x)} n_x + \underset{\sim}{\tau}^{(y)} n_y + \underset{\sim}{\tau}^{(z)} n_z$$

$$\tau_i = \tau_i^{(j)} n_j$$

$$\underset{\sim}{\tau} = \underset{\approx}{T} \cdot \underset{\sim}{n} \qquad T_{ij} = \tau_j^{(i)}$$

$$\underset{\approx}{T} = \begin{bmatrix} \tau_x^{(x)} & \tau_y^{(x)} & \tau_z^{(x)} \\ \tau_x^{(y)} & \tau_y^{(y)} & \tau_z^{(y)} \\ \tau_x^{(z)} & \tau_y^{(z)} & \tau_z^{(z)} \end{bmatrix}$$

T_{xy}

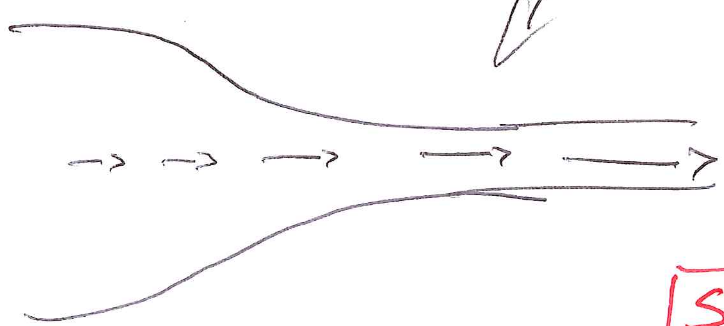
II) what is $\rho\left(\frac{\partial v}{\partial t} + v \cdot \nabla v\right)$?

REMIND THIS IS JUST $\frac{dv}{dt}$ = ACC. PART.

$$v(t, x(t)) \Rightarrow \frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} v_x + \frac{\partial v}{\partial y} v_y + \frac{\partial v}{\partial z} v_z$$

ACCEL.

(ON THE RHS ARE FORCES f & σ)



$\boxed{\text{S19-S20}}$

Eq. Cons. Momento QdM

$\Rightarrow \mathbb{T}$ è simmetrico

FINE

Eq. cons. QdM - Eq CAUCHY

$$\begin{cases} \dfrac{\partial v}{\partial t} + v \cdot \nabla v = -\dfrac{1}{\rho} f + \dfrac{1}{\rho} \nabla \cdot \mathbb{T} \\ \nabla \cdot v = 0 \end{cases}$$

4 EQ. ; INC. 3 (v) + 6 ($\mathbb{T}$) = 9! we miss something

what is $\mathbb{T}$?

IDROSTATICA $\quad \bar{\tau} = p\,\bar{n} = \bar{\bar{\mathbb{T}}} \cdot \bar{n}$

$$\mathbb{T} = p\,\mathbb{I} = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}$$

$$\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} = \frac{\partial p}{\partial x} \quad ; \quad \nabla \cdot \mathbb{T} = \nabla p$$

To HAVE 4 INCOGNITE

ONLY DEF.

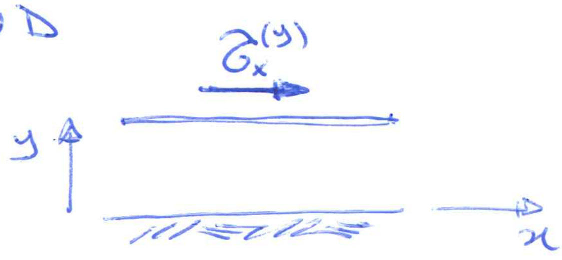
$$\mathbb{T} = p\,\mathbb{I} + f(\nabla \bar{v}) \overset{\text{STOKESIAN FLUID}}{=\!=\!=} p\,\mathbb{I} + f(\mathbb{D})$$

viscosity enters here

$$\underset{\approx}{\mathbb{T}} = p\,\mathbb{I} + f_\mu\left(\text{derivate } \underset{\sim}{v}\right)$$

REMIND

NEWTONIAN FLUID



$$\tau_x^{(y)} = T_{xy} = -\mu \frac{\partial v_x}{\partial y}$$

simmetrico e scambiando y con x $T_{yx} = -\mu \frac{\partial v_y}{\partial x}$

IN GEN

$$T_{xy} = -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$\underset{\approx}{\mathbb{T}} = \begin{bmatrix} p - 2\mu \frac{\partial v_x}{\partial x} & -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) & -\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) \\ \#\text{sym} & p - 2\mu \frac{\partial v_y}{\partial y} & -\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) \\ \# & \# & p - 2\mu \frac{\partial v_z}{\partial z} \end{bmatrix}$$

EQ. FLUIDI NEWTONIANI

$$\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{xy}}{\partial y}+\frac{\partial T_{xz}}{\partial z}=\frac{\partial p}{\partial x}-\cancel{2\mu\frac{\partial^2 v_x}{\partial x^2}}-\cancel{\mu\frac{\partial^2 v_x}{\partial x\partial y}}-\mu\frac{\partial^2 v_x}{\partial x^2}-\cancel{\mu\frac{\partial^2 v_z}{\partial x\partial z}}-\mu\frac{\partial^2 v_x}{\partial x^2}$$

$$=\frac{\partial p}{\partial x}-\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)=\frac{\partial p}{\partial x}-\mu\nabla^2 v_x$$

$$\begin{cases}\dfrac{\partial v_x}{\partial t}+v_x\dfrac{\partial v_x}{\partial x}+v_y\dfrac{\partial v_x}{\partial y}+v_z\dfrac{\partial v_x}{\partial z}=-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}+\dfrac{\mu}{\rho}\nabla^2 v_x+f_x\\ \vdots\end{cases}$$

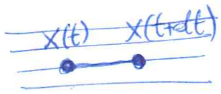
$$\begin{cases}\overbrace{\dfrac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}}^{\text{ACC. PARTIC. MATERIALE}}=\overbrace{\underset{\sim}{f}-\dfrac{1}{\rho}\nabla p}^{\text{F. APPL.}}+\overbrace{\nu\nabla^2\underset{\sim}{v}}^{\text{F. ATTRITI}}\\ \nabla\cdot\underline{v}=0\end{cases}$$

$\ldots\ \underset{\sim}{a}=\dfrac{F}{m}$

NAVIER-STOKES

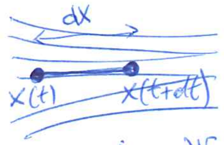
MID '800

MOTO UNIF. $v(t)$



$a=\dfrac{\partial v}{\partial t}$

MOTO STAZ. $v(x)$



$$a=\frac{\left(v+\frac{\partial v}{\partial x}dx\right)-v}{dt}=\frac{\partial v}{\partial x}\frac{dx}{dt}=v\frac{\partial v}{\partial x}\quad \text{CONVEZ}$$

Attriti

perdite di energia in calore

Eq Navier-Stokes

$$\frac{\partial v}{\partial t} + v\cdot\nabla v = \frac{1}{\rho} f - \frac{1}{\rho}\nabla p + \nu \nabla^2 v$$

- viscosity is small, dissip. are small
- neglecting viscosity → EULER EQ.

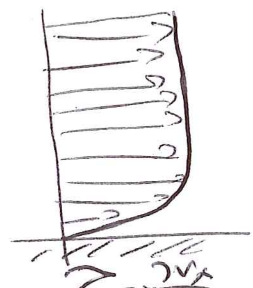
$$\frac{\partial v}{\partial t} + v\cdot\nabla v = \frac{1}{\rho} f - \frac{1}{\rho}\nabla p$$ IDEAL FLUIDS frictionless

- usable shor paths
- far from walls
- reversible $t \to -t$, $v \to -v$
- 1^{st} ORDER EQ. ⇒ 1 b.c.

- 2^{nd} ORDER EQ.

b.c. $v_n = 0$ impermeable

$v_{tg} = 0$ aderence



$\tau = \mu \frac{\partial v_x}{\partial y}$

- Moto mostly unidirectional $v_x \gg v_y, v_z$

$\frac{\partial v_y}{\partial t} = \frac{\partial v_y}{\partial x} = 0$

$v_y \equiv 0 \Rightarrow \frac{\partial p}{\partial y} = 0$ transv. press. is constant

$v_z \equiv 0 \Rightarrow \frac{\partial p}{\partial z} + g = 0$ $p = -gz$ IDROSTAT.

S21 - S22

6 Bernoulli Balance

Eq. NS

HP No dissip.

HP $\bar{f} = -\nabla g z$

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla\left(\frac{p}{\rho} + g z\right)$$

forma alternativa modif. $v \cdot \nabla v$

$$v \cdot \nabla v\Big]_x = \underbrace{v_x \frac{\partial v_x}{\partial x}} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} + \left[\underbrace{v_y \frac{\partial v_y}{\partial x}} - \underbrace{v_y \frac{\partial v_y}{\partial x}} + \underbrace{v_z \frac{\partial v_z}{\partial x}} - v_z \frac{\partial v_z}{\partial x}\right]$$

$$= \frac{1}{2}\frac{\partial}{\partial x}(v_x^2 + v_y^2 + v_z^2) + v_y \underbrace{\left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}\right)}_{-\omega_z} + v_z \underbrace{\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)}_{+\omega_y} =$$

$$= \frac{1}{2}\frac{\partial v^2}{\partial x} - (v_y \omega_z - v_z \omega_y) = \nabla \frac{v^2}{2}\Big]_x - [\bar{v} \times \omega]_x$$

$$\frac{\partial v}{\partial t} + \nabla\left(\frac{v^2}{2} + \frac{p}{\rho} + g z\right) = \bar{v} \times \bar{\omega}$$

HP proj linea di corrente $\bar{s} = \frac{\bar{v}}{|v|}$

$$\frac{\partial v_s}{\partial t} + \frac{\partial}{\partial s}\left(\frac{v^2}{2} + \frac{p}{\rho} + g z\right) = 0$$

Alternatively (same hp) less rigorous but intuitive

consider local "Cartesian" coordinates aligned with a (sufficiently straight) streamline

$$\Rightarrow \quad \frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{\partial \hat{z}}{\partial s} \qquad v = \|v\| = \|v_s\|$$

$\boxed{S23}$

$$\frac{\partial v_s}{\partial t} + \frac{\partial}{\partial s}\left(\frac{v^2}{2} + \frac{p}{\rho} + gz\right) = 0$$

$\int_1^2$ between two points

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + \int_1^2 \frac{\partial v_s}{\partial t}\, ds$$

$$H = z + \frac{p}{\gamma} + \frac{v^2}{2g} \quad ; \quad \gamma H = \underbrace{\gamma z + p}_{\text{pot. energy}} + \underbrace{\rho \frac{v^2}{2}}_{\text{kin energy}}$$

(per unit volume)

$$H_1 = H_2 + \int_1^2 \frac{\partial v_s}{\partial t}\, ds$$

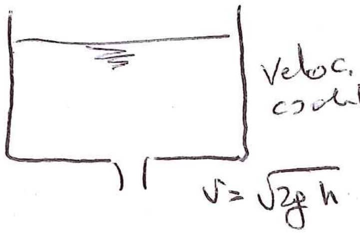
conserv. of mech energy

(consequence of cons. of QdM in potential incompressible)

$\boxed{S24}$

HP $\frac{\partial v}{\partial t} = 0$ CASO STAZIONARIO

$$H = \frac{v^2}{2} + \frac{p}{\rho} + gz = \text{costante lungo linea di corr.}$$

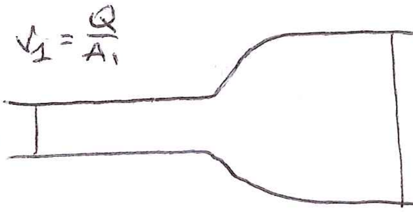


Veloc. costante

$v = \sqrt{2gh}$

S25

$v_1 = \frac{Q}{A_1}$ A_1



A_2

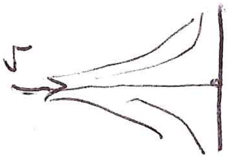
$v_2 = \frac{Q}{A_2} < v_1$

$\Downarrow$

$p_2 > p_1$

$$p_1 - p_2 = \frac{\rho}{2}(v_1^2 - v_2^2) = \rho\frac{Q^2}{2}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right)$$

recupero press. sopra stenotica!

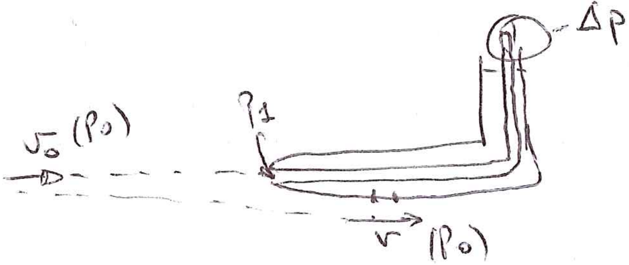


splash
(recupero press. x impatto

$\Delta p = \rho\frac{v^2}{2}$

S26

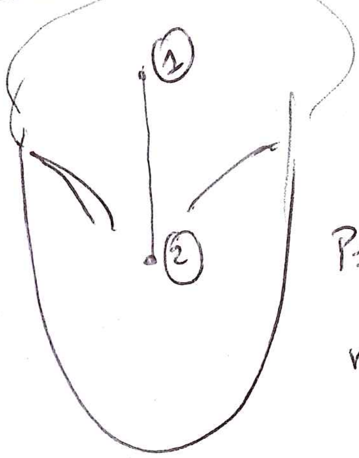
Pitot



$v_0\ (p_0)$ p_1 Δp $v\ (p_0)$

$$v = \sqrt{2\frac{\Delta p}{\rho}}$$

ALSO $\frac{\partial v}{\partial t} = 0$ At PEAK of pulsation



$$p_1 + \rho\frac{v_1^2}{2} = p_2 + \rho\frac{v_2^2}{2}$$

$v_1 \ll v_2$

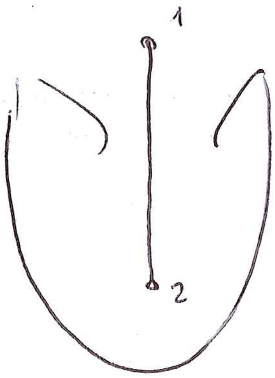
TRANSV. Δp

S27

$$\Delta p = \frac{\rho}{2} v_2^2 \qquad \Delta p_{[Pa]} = \frac{1050}{2} v^2_{[m/s]} = 133 \times \Delta p_{[mmHg]}$$

$$\Delta p_{mmHg} = \frac{1050}{133 \times 2} v^2_{[m/s]} \simeq 4v^2$$

MORE IN GENERAL $\frac{\partial v}{\partial t} \neq 0$



$$\frac{\partial v_s}{\partial t} + \frac{\partial}{\partial s}\left(\frac{p}{\rho} + \frac{v^2}{2}\right) = 0$$

$$\int_1^2 \left[\frac{\partial v_s}{\partial t} + \frac{\partial}{\partial s}(\ )\right] ds = 0$$

$\left(\tilde{p} = p + \gamma z\right)$

$$p_1 + \rho\frac{v_1^2}{2} = p_2 + \rho\frac{v_2^2}{2} + \underbrace{\int_1^2 \frac{\partial v_s}{\partial t} ds}_{} \text{— INERTIA}$$

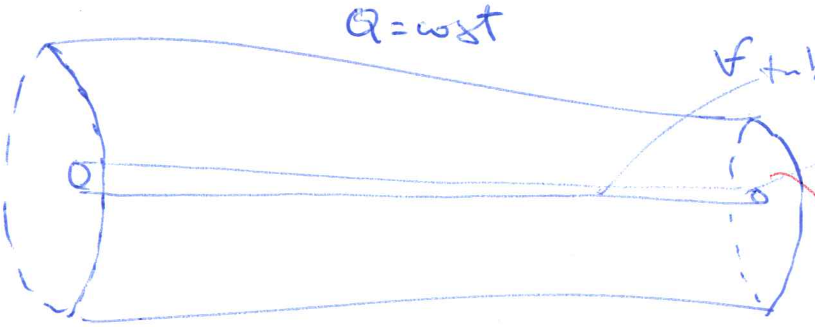
$$p_1 - p_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) + \int_1^2 \frac{\partial v_s}{\partial t} ds$$

$$p_1 - p_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) + \frac{\partial \overline{v}_s}{\partial t} L_{12}$$

$\Uparrow$

ACCELERAZ.

Estensione BERNOULLI x CORRENTI



POTENZA di UNA SEZIONE DI FLUSSO

$$P \triangleq \int_A \gamma H \, dQ$$ costante lungo la corrente

$$P \triangleq \int \gamma H \, dQ = \gamma \int H v \, dA = \gamma \int h v \, dA + \gamma \int \frac{v^3}{2g} dA =$$

$$= \gamma h Q + \gamma \alpha \frac{V^3}{2g} A = \gamma h Q + \gamma \alpha \frac{V^2}{2g} Q =$$

$$\alpha \triangleq \frac{\int v^3 dA}{V^3 A} \simeq 1$$ ma può essere molto diverso

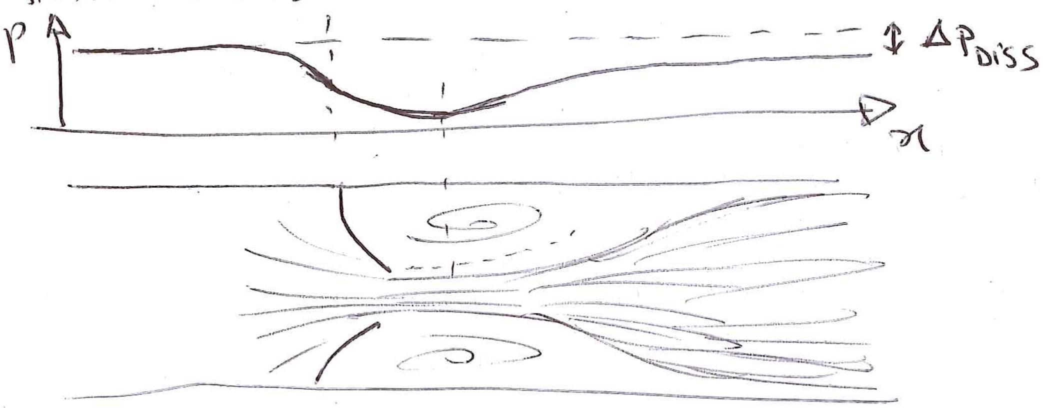
$$= \gamma Q \left(h + \alpha \frac{V^2}{2g} \right)$$

$$\boxed{\bar{H} = h + \alpha \frac{V^2}{2g}}$$ costante lungo la corrente

Esteg. Bernoulli x Dissip 27.

current ΔP — GRAVIT — OK
— $\frac{\partial}{\partial t} = 0$ if needed OK
— IDEAL ?? Across valves not much

ANDAM PRESSIONI

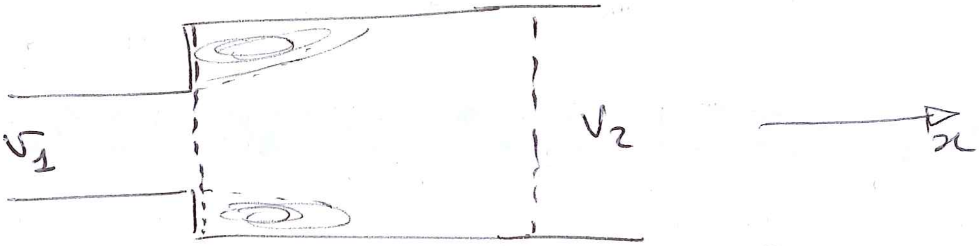


even in the Stat. case

$p + \rho \frac{v^2}{2}$ is NOT constant there is a ΔP_{DISS}.

$$p_1 + \rho \frac{v_1^2}{2} = p_2 + \rho \frac{v_2^2}{2} + \underbrace{\Delta P_{DISS}}_{\text{LOSS}} + \underbrace{\rho \frac{\partial \bar{v}}{\partial t} L_{12}}_{\text{INERTA}}$$

$$\Delta P_{DISS} = p_1 + \rho \frac{v_1^2}{2} - p_2 - \rho \frac{v_2^2}{2} - \rho \frac{\partial \bar{v}}{\partial t} L_{12}$$



$$I_x + M_x = \Pi_x + \cancel{G}$$

$$M = \rho \int_A V_x (\bar{v} \cdot \bar{n})\, dA = -\rho V_1^2 A_1 + \rho V_2^2 A_2$$

$$I = \rho \int_{A_2 \times L} \frac{\partial V_x}{\partial t}\, dV = \rho \int_L \int_{A_2} \frac{\partial v}{\partial t}\, dA\, dx = \rho \frac{\partial V_2}{\partial t} A_2 L$$

$$\Pi = P_1 A_2 - P_2 A_2$$

QdM $\Rightarrow$ $$\rho \frac{\partial V_2}{\partial t} \cancel{A_2} L + \rho V_2^2 \cancel{A_2} - \rho V_1^2 \frac{A_1}{A_2} = (P_1 - P_2) \cancel{A_2}$$

$$P_1 - P_2 - \rho \frac{\partial \bar{V}}{\partial t} L = \rho V_2^2 - \rho V_1^2 \frac{A_1}{A_2}$$

BY DEF: $$\Delta P_{LOSS} = P_1 - P_2 - \rho \frac{\partial \bar{V}}{\partial t} L + \rho \frac{V_1^2}{2} - \rho \frac{V_2^2}{2} =$$

$$= \rho V_2^2 - \rho V_1^2 \frac{A_1}{A_2} + \rho \frac{V_1^2}{2} - \rho \frac{V_2^2}{2} =$$

$$= \rho \frac{V_2^2}{2} - \rho \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right) = \rho \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2 = \rho \frac{V_2^2}{2}\left(\frac{A_2}{A_1} - 1\right)^2$$

IF $A_2 >> A_1$ $\quad \Delta P_{LOSS} \simeq \rho \frac{V_1^2}{2}$ All KE INLET

in gen $\quad \Delta P_{LOSS} = \alpha \rho \frac{V^2}{2}$

7 MOTO IN CONDOTTI RETTILINEI

STRATO LIMITE

$\boxed{S28}$

$$NS_x) \quad \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \underbrace{\left\{ \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right\}}_{\text{Viscous Term.}}$$

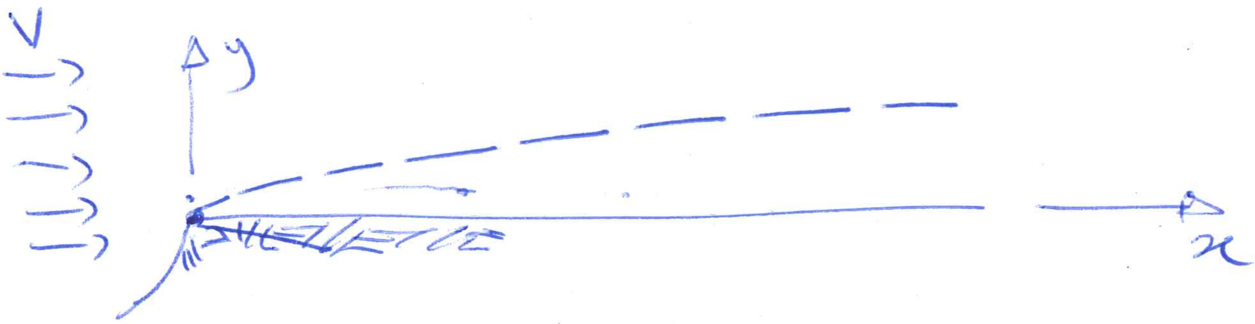
* Visc. term is SMALL
  - Visc. Term NEGLIGIBLE "locally" (Bernoulli)
  - Relevant along "long paths"

$\boxed{S29}$

BUT Visc. term unavoidable near the wall because b.c. aderenza

THUS:

Moto sup. piane (inlet of a vessel)



DEF. strato limite = lo spessore vicino alla parete in cui il t. visc. è ≈ con altri Termini

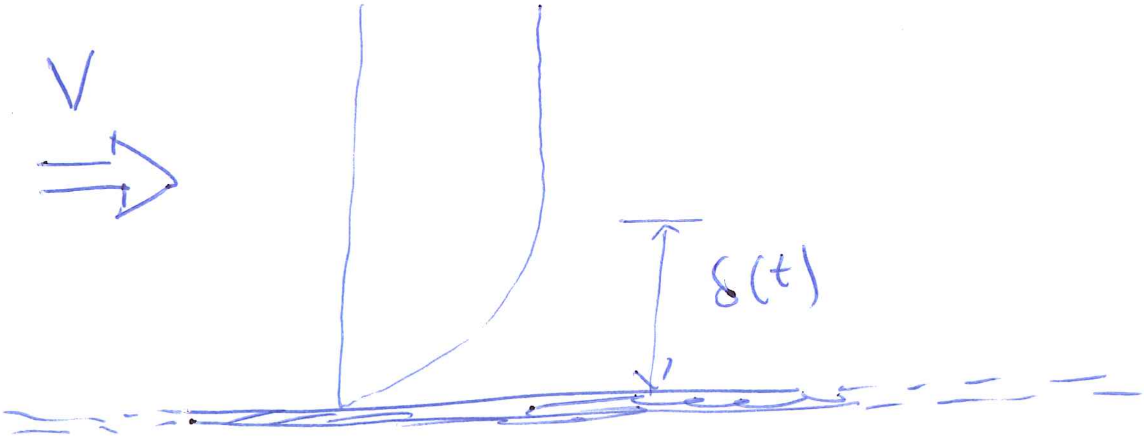
$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}} + \frac{\partial^2 v_x}{\partial y^2}\right)$$

$$\frac{V^2}{x} \approx v_y \frac{V}{\delta} \approx \qquad \cancel{\nu\frac{V}{x^2}} \qquad \nu\frac{V}{\delta^2}$$

v_y: CONT. $\frac{V\delta}{x}$

$$\delta(x) \sim 5\sqrt{\frac{\nu x}{V}}$$

IMPULSIVELY STARTED FLOW



$$v=0 \quad \Rightarrow \quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \Rightarrow \quad u(y,t)$$

$$\frac{\partial p}{\partial y}=0 \qquad p(x,t) \qquad \text{ma} \quad p(+\infty)=p(-\infty) \Rightarrow \frac{\partial p}{\partial x}=0$$

$$\frac{\partial u}{\partial t}=\nu \frac{\partial^2 u}{\partial y^2}$$ PARABOLIC : Eq. Diffusione

SELF-SIMILAR $$u(y,t)=f\left(\frac{y}{\delta(t)}\right)$$

$$\frac{\partial u}{\partial t}=-f'\frac{y}{\delta^2}\delta'=-\frac{y}{\delta^2}f'\delta'$$

$$\frac{\partial u}{\partial y}=f'\frac{1}{\delta} \quad ; \quad \frac{\partial^2 u}{\partial y^2}=f''\frac{1}{\delta^2}$$

$$\Rightarrow \quad -\frac{1}{\delta}\frac{d\delta}{dt}\frac{y}{\delta}f'=\frac{\nu}{\delta^2}f''$$

$$-\frac{\delta}{\nu}\frac{d\delta}{dt}=\frac{f''\left(\frac{y}{\delta}\right)}{\frac{y}{\delta}f'\left(\frac{y}{\delta}\right)}=\text{cost}=-K$$

$$\delta \frac{d\delta}{dt} = \nu K$$

$$\frac{1}{2}\frac{d\delta^2}{dt} = \nu K \Rightarrow \delta^2 = 2K\nu t + \cancel{\text{const}}^{\,0} \qquad \delta(0)=0$$

$$\delta = \sqrt{2K\nu t} \overset{K=2}{=\!=} 2\sqrt{\nu t}$$

by dim. arguments $\boxed{S30}$

GROWTH VEL. $\frac{d\delta}{dt} = f(\delta, \nu)$ not by V that gives velocity of b.l.

$$\frac{d\delta}{dt} \approx \frac{\nu}{\delta} \Rightarrow \delta \approx \sqrt{\nu t}$$

CASE STEADY

$\delta(x)$

if observer stay fixed at $t=0$ at the tip. $x=0$
the plate moves t at $x = Vt$

$$\delta \sim \sqrt{\nu t} \Rightarrow \delta \sim \sqrt{\frac{\nu x}{V}}$$

~~CONTROLLO coefficienti 2V esatto (SAHS)~~

DIFFUSION PHENOMENON

PARABOLIC PART of NS. Eq.

in a transient of duration T $\qquad \delta_{MAX} \approx \sqrt{\nu T}$

in a brief length L $\qquad \delta_{MAX} \approx \sqrt{\frac{\nu L}{V}}$

b.l. is the region where <u>blood flow</u> interacts with <u>vessel tissue</u> (epitelio)

interaction is through "forces" $\tau_{tg(wall)} = WSS$

$$\tau_w = WSS = \frac{\partial V_{tg}}{\partial n}$$

quantification of WSS for atherosclerosis

epitelio made of elongated ordered cells which may be altered by not large+longitudinal WSS

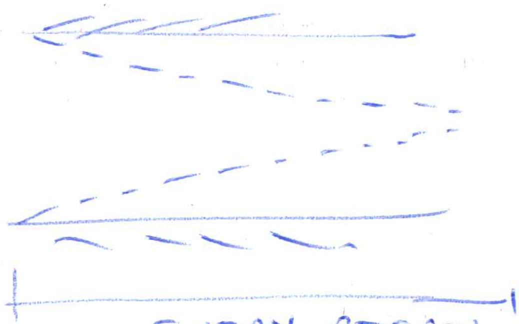
low WSS   oscillating WSS   large WSS gradient

correlate with

aterosclerosys

Looking at vessels try to read WSS pattern

Condotto



$\delta_x \simeq D$

$$\delta(x) = D \qquad D^2 = c^2 \frac{\nu x}{V} \Rightarrow \frac{x}{D} \simeq \frac{1}{c^2}\frac{VD}{\nu}$$

~~AORTA~~

~~AOR~~	V	D	$\frac{x}{D}$	
AORTA	50 cm/s	3 cm	100 - 200	never uniform
MID ARTERY	10 cm/s	~~0,5~~ 1 cm	~~8~~ 10	
SMALL ARTERIOL	5 cm/s	0,25 cm	1	always uniform
ARTERIOLE	0,1	<0,1 cm	< 1	

SOLUZIONI DI
MOTO UNIDIREZ.

1

Scorrimento

HP $v_y = v_z = 0$ UNIDIREZ.

$\frac{\partial}{\partial t} = 0$ STAZ.

$y, z \rightarrow p + \gamma z = \text{cost.}$

contin. $\frac{\partial v_x}{\partial x} = 0$ UNIDIREZ. $\Rightarrow$ UNIFORME

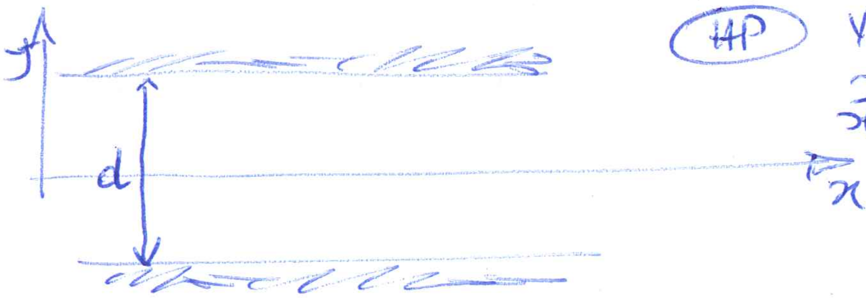
x) HP $\frac{\partial p}{\partial x} = 0 \Rightarrow \frac{\partial^2 v_x}{\partial y^2} = 0$

$v_x(y) = Ay + B$ + b.c. $v_x(0) = 0$ $v_y(d) = V$

$B = 0$ $A = \frac{V}{d}$

$v_x(y) = V \frac{y}{d}$ $\quad \tau = \tau_w = \frac{V}{d}$

Flusso tra 2 lamine



HP $v_y = v_z = 0$ UNID.

$\frac{\partial}{\partial t} = 0$ STAZ.

S33

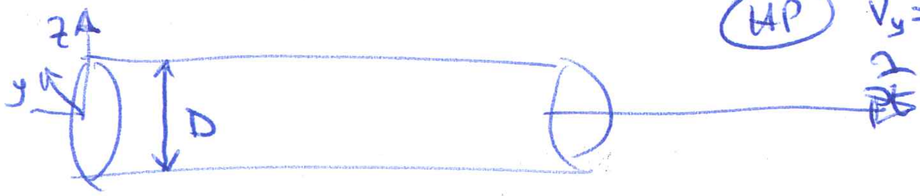
$y, z \rightarrow$ IDROST. ; $\frac{\partial v_x}{\partial x} = 0$

forzante $\rightarrow$ $\overbrace{\frac{1}{\rho}\frac{\partial p}{\partial x}}^{K} = \nu \frac{\partial^2 v_x}{\partial y^2}$ b.c. $v_x(\pm d) = 0$

$v_x(y) = \frac{K}{2\nu}\left(\frac{d^2}{4} - y^2\right)$

$\tau(y) = -\frac{K}{2\nu} y$; $\tau_w = -\frac{Kd}{2\nu}$

Flusso in vaso cilindrico



HP $V_y = V_z = 0$ UNIDIREZ.

$\frac{\partial}{\partial t} = 0$ STAZ.

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\left(\frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial y^2}\right)$$

coord. cilindriche $x, y, z \to x, r, \theta$

(*) x simmetria $\frac{\partial}{\partial\theta} = 0$ (ASSIALSIMMETRICO)

$V_x(r)$

$$\frac{1}{\rho}\frac{\partial p}{\partial x}\; K = \nu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_x}{\partial r}\right) \quad + \text{b.c. } V_x(D/2) = 0 \;\&\; V_x < \infty$$

$$V_x(r) = \frac{K}{4\nu}\left(R^2 - r^2\right)$$

$$\tau(r) = -\frac{K}{2\nu} r \qquad \tau_w = -\frac{KR}{2\nu}$$

$$Q = 2\pi\int_0^R V_x\, r\, dr = \frac{\pi}{128}\frac{KD^4}{\nu} \; ; \; V = \frac{Q4}{\pi D^2} = \frac{KD^2}{32\nu}$$

$$V_x(r) = 2V\left(1 - \left(\frac{r}{R}\right)^2\right)$$

$$r = \sqrt{x^2+y^2} \qquad \frac{\partial r}{\partial x} = \frac{x}{r} \qquad \frac{\partial r}{\partial y} = \frac{y}{r}$$

$$\vartheta = \tan^{-1}\frac{y}{x} \qquad \frac{\partial^2 r}{\partial x^2} = \frac{1}{r} - \frac{x^2}{r^3} \qquad \frac{\partial^2 r}{\partial y^2} = \frac{1}{r} - \frac{y^2}{r^3}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r}\frac{\partial r}{\partial x}$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial r}{\partial x}\frac{\partial}{\partial r}\right) = \frac{\partial^2 r}{\partial x^2}\frac{\partial}{\partial r} + \left(\frac{\partial r}{\partial x}\right)^2\frac{\partial^2}{\partial r^2}$$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \left(\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2}\right)\frac{\partial}{\partial r} + \left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]\frac{\partial^2}{\partial r^2} =$$

$$= \left(\frac{2}{r} - \frac{1}{r}\right)\frac{\partial}{\partial r} + \left(\frac{x^2}{r^2} + \frac{y^2}{r^2}\right)\frac{\partial^2}{\partial r^2} =$$

$$= \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$$

COEFF. RAGGUAGLIO PER Poiseuille

$$V_x(r) = 2V\left(1-\left(\frac{r}{R}\right)^2\right)$$

$$\beta = \frac{\int v_x^2 \, dA}{V^2 \cancel{A}A} = \frac{4\cancel{V^2}\,2\cancel{\pi}}{\cancel{V^2}\cancel{\pi}R^2}\int_0^R \left(1-\left(\frac{r}{R}\right)^2\right)^2 \cancel{r}\, dr =$$

$$x = \frac{r}{R};$$

$$= 8\int_0^1 (1-x^2)^2 x \, dx = 8\int_0^1 (x + x^5 - 2x^3)\, dx =$$

$$= 8\left[\frac{x^2}{2} + \frac{x^6}{6} - \cancel{2}\frac{x^4}{\cancel{4}2}\right]_0^1 = 8 \cdot \frac{1}{6} = \frac{4}{3}$$

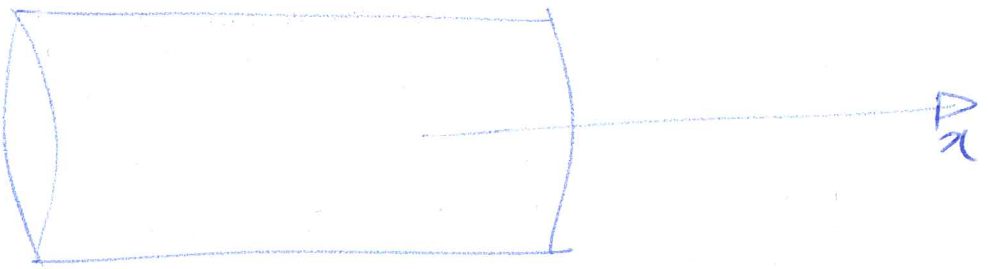
$$\alpha = \frac{\int v_x^3 \, dA}{V^3 \cancel{A}} = \frac{8\cancel{V^3}\,2\cancel{\pi}}{\cancel{V^3}\cancel{\pi}\cancel{R^2}}\int (1-x^2)^3 x \, dx =$$

$$= 16\int_0^1 (1 - 3x^2 + 3x^4 - x^6)\, x \, dx = 16\int_0^1 (x - 3x^3 + 3x^5 - x^7)\, dx$$

$$= 16\left[\frac{x^2}{2} - \frac{3x^4}{4} + \cancel{3}\frac{x^6}{\cancel{6}2} - \frac{x^8}{8}\right] = 16\left[\frac{1}{2} - \frac{3}{4} + \frac{1}{2} - \frac{1}{8}\right] =$$

$$= 16\,\frac{4-6+4-1}{8} = 2$$

OSCILLATORY FLOW

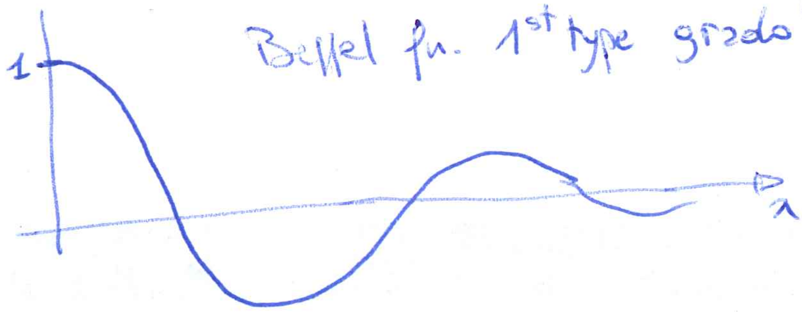


$$\begin{cases} \dfrac{\partial v_x}{\partial t} + \underbrace{K \sin \omega t}_{\frac{1}{\rho}\frac{\partial p}{\partial x} \text{ FORCING}} = \nu \dfrac{1}{r} \dfrac{\partial}{\partial r}\left(r \dfrac{\partial v_x}{\partial r} \right) \\ \\ v_x(R) = 0 \end{cases}$$

SOLUTION:

$$v_x(t,r) = \frac{K}{\omega}\left[1 - \frac{J_0\left(\sqrt{-i\frac{\omega}{\nu}}\, r\right)}{J_0\left(\sqrt{-i\frac{\omega}{\nu}}\, R\right)} \right] e^{i\omega t}$$

$J_0(x)$ Bessel fn. 1st type grado "0"



$$\underline{v_x\left(\omega t, \frac{r}{R}\right)} = \overset{V}{\frac{K}{\omega}} \left[1 - \frac{J_0\left(\alpha \sqrt{-i}\, \frac{r}{R}\right)}{J_0\left(\alpha\sqrt{-i}\right)} \right] e^{i\omega t}$$

$$\alpha = R\sqrt{\frac{\omega}{\nu}} = \left(\frac{R^2 \omega}{\nu}\right)^{1/2} = \left(\frac{R^2 2\pi}{\nu T}\right)^{1/2}$$

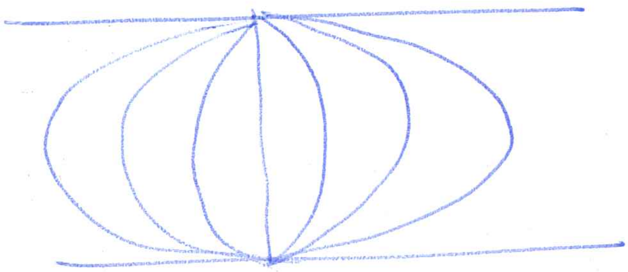
~~AORTA [illegible] 78~~

~~α = 20 POISUELLE MODIFICATO~~ ~~[illegible]~~

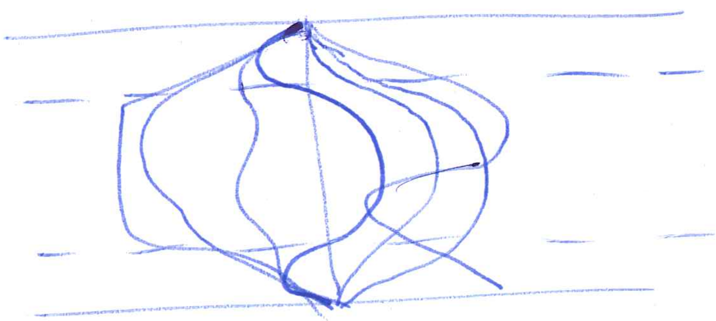
$$\alpha \approx \frac{D}{\sqrt{\nu T}} \quad \begin{matrix} \leftarrow \text{diameter} \\ \leftarrow \delta_{max} \end{matrix}$$

α small $\Rightarrow \delta \gg D \Rightarrow \delta = \frac{D}{2}$

successione di moti stazionari



α large $\Rightarrow \delta \ll D$ no time to diffuse



$\updownarrow \delta \sim \sqrt{\nu T}$

$$Re = \frac{VD}{\nu} \qquad St = \frac{D}{VT} = \frac{\alpha^2}{Re}$$

	V (cm/s)	D (cm)	Re	α	St
AORTA	50	3	4000	20	0,05
MID ARTERIE	10	1	300	6	0,1
SMALL "	5	0,25	40	1,5	0,05
ARTERIOLE	0,1	<0,1	<1	0,5	~1

PULSATILE FLOW

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = K_0 + K_1 \sin\omega t + \ldots$$

$$v_x\left(t, \frac{r}{R}\right) = \underbrace{\frac{K_0 R^2}{4\nu}\left(1 - \left(\frac{r}{R}\right)^2\right)}_{\text{Poiseuille}} + \underbrace{\frac{K_1}{\omega}\left[1 - \frac{J_0\left(\alpha\sqrt{-i}\,\frac{r}{R}\right)}{J_0(\alpha\sqrt{-i})}\right]}_{\text{Amplitude } \mathbb{C}} e^{i\omega t} + \ldots$$

$2V_0$

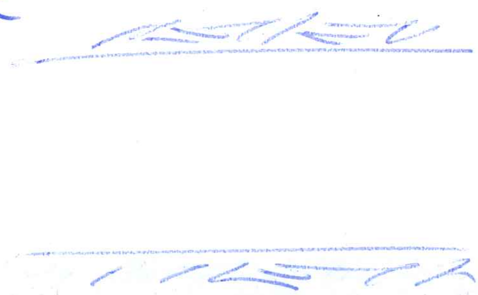
α small

succession of steady solutions

$\delta = \frac{D}{2}$

S34 - S35

α large



$$W = \frac{D}{\sqrt{\nu T}}$$

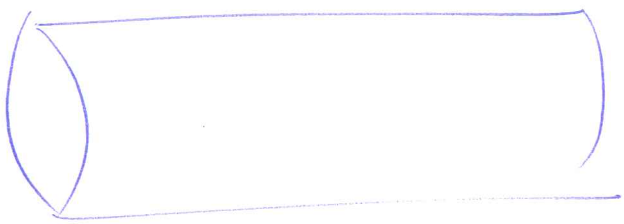
$\boxed{S36}$

$$Re = \frac{VD}{\nu}$$

$$St = \frac{D}{VT}$$

MOTO QUASI UNIDIREZIONALE
MOTO TURBOLENTO

Esper. Re



$Re_{cr} \approx 2500$
Instabilità moto laminare
sol. NS. instabile
∃ altre soluzioni

Moto apparentemente casuale ma NS deterministico
"stocasticizzazione" delle soluzione

Ad esempio chaos $x^{t+1} = \lambda x^t (1-x^t)$ deterministica ma caotica
$\boxed{1D} T$ S.I.C.

Stesso pb. con NS. sol. NUMERICA?

⟹ GRIGLIA DI CALCOLO

L dim max $\quad \eta$ dim minima = ?

SMALL SCALES
SPETTRI

$$\epsilon = \frac{dV^2}{dt} \sim \frac{V^2}{T} = \frac{V^3}{L}$$

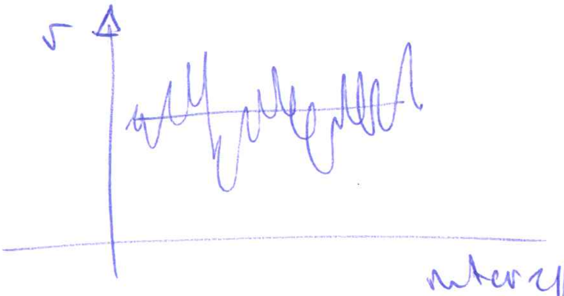
$$\eta = f(\epsilon, \nu) \qquad [L] = \frac{\nu^{3/4}}{\epsilon^{1/4}}$$

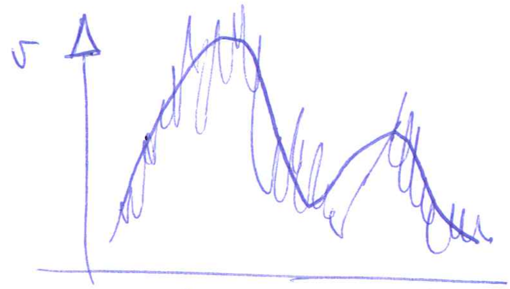
$$\eta = K \frac{\nu^{3/4}}{\epsilon^{1/4}}$$

$$N = \frac{L}{\eta} \sim \frac{L}{\nu^{3/4}} \epsilon^{1/4} = Re^{3/4}$$

IRRISOLVIBILE

TBL ?? Big mistery of class. mech.





interesse in moto medio!

$$\langle \bar{v} \rangle = \frac{1}{T}\int_{-T/2}^{T/2} v(x,t)\,dt$$

(o medie spaziale
filtri vari...)

$$v' = v - \langle v \rangle$$

Eq. Moto medio

CONT. $\langle \nabla \cdot v \rangle = 0 \qquad \nabla \cdot \langle v \rangle = 0 \Rightarrow \nabla \cdot v' = 0$

N.S.

$$\frac{\partial \langle v_x \rangle}{\partial t} + \langle \bar{v} \cdot \nabla v_x \rangle = -\frac{1}{\rho}\frac{\partial}{\partial x}\langle p \rangle + \nu \nabla^2 \langle v_x \rangle$$

$$\left\langle (\langle v_x \rangle + v_x') \frac{\partial(\langle v_x \rangle + v')}{\partial x} + (y)(z) \right\rangle = \langle \bar{v} \rangle \cdot \nabla \langle v_x \rangle + \langle \langle \bar{v} \rangle \cdot \overbrace{\nabla v_x'}^{=0} \rangle + \langle\ \rangle +$$

$$+ \left\langle v_x' \frac{\partial v_x'}{\partial x} + v_y' \frac{\partial v_x'}{\partial y} + v_z' \frac{\partial v_x'}{\partial z} \right\rangle + \underbrace{\left\langle v_x' \cdot \frac{\partial v_x'}{\partial x} + v_x' \frac{\partial v_y'}{\partial y} + v_x' \frac{\partial v_z'}{\partial z} \right\rangle}_{\text{Aggiungo } v_x' \nabla v_x'} =$$

$$= \langle \bar{v} \rangle \cdot \nabla \langle v_x \rangle + \frac{\partial}{\partial x}\langle v_x' v_x' \rangle + \frac{\partial}{\partial y}\langle v_x' v_y' \rangle + \frac{\partial}{\partial z}\langle v_x' v_z' \rangle$$

$$\Pi_{R_{ij}} = -\langle v_i' v_j' \rangle$$

$$\frac{\partial \langle \bar{v} \rangle}{\partial t} + \langle \bar{v} \rangle \cdot \nabla \langle \bar{v} \rangle = -\frac{1}{\rho}\nabla \langle p \rangle + \nu \nabla^2 \langle \bar{v} \rangle + \underline{\nabla \cdot \Pi_R}$$

Tensori di stress
effetti small scales TBL
su moto medio

Closure pb

modelli $\Pi_R = \nu_{TTBL}\left(\frac{\partial \langle v_i \rangle}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ Boussinesq

MOTO "MEDIAMENTE" UNIDIREZIONALE

HP $\langle v_y \rangle = \langle v_z \rangle = 0$

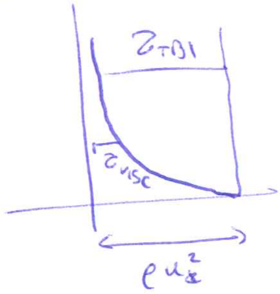
$\Rightarrow \langle v_x \rangle (y)$

HP $\frac{\partial}{\partial x}\langle p \rangle = 0$

in realtà non importa per come è il moto solo per il valore delle velocità

eq. Re (x) $\quad \nu \frac{\partial^2 \langle v_x \rangle}{\partial y^2} - \frac{\partial \langle v_x' v_y' \rangle}{\partial y} = 0$

$$\Rightarrow \nu \frac{\partial \langle v_x \rangle}{\partial y} - \langle v_x' v_y' \rangle = u_*^2$$



MOLTO VICINO PARETE

$$\langle v_x \rangle = f(u_*, \nu, y)$$

$$[V] = u_*$$

$$[L] = \frac{\nu}{u_*}$$

$\langle v_x \rangle = u_* \cdot \frac{y u_*}{\nu}$ LINEAR

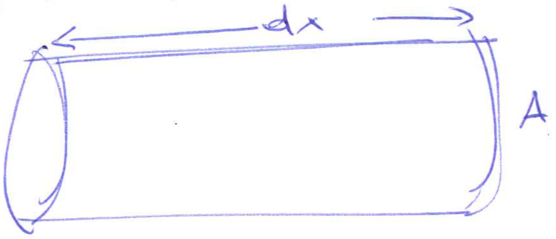
MOLTO LONTANO POSSO TRAS. ν se $y >> \frac{\nu}{u_*}$

$\langle v_x \rangle = ?$ c'è la b.c. dep. on ν

$$\underbrace{\frac{\partial \langle v_x \rangle}{\partial y}}_{\text{LOCAL}} = f(u_*, y) = \frac{1}{K} \frac{u_*}{y}$$

$\Rightarrow \langle v_x \rangle = \frac{u_*}{K} \ln \frac{y}{y_0}$ PIATTO!! VALE SEMPRE!!

1. cos'è u_*?



$$\cancel{G + \Pi = M_u - M_e} \Rightarrow \tau_w \cdot C = -\frac{\partial p}{\partial x} A \qquad u_*^2 = -\frac{\partial p}{\partial x}\frac{A}{C} = -\frac{\partial p}{\partial x}\frac{D}{4}$$

K

$$u_* = \sqrt{-\frac{D}{4\rho}\frac{dp}{dx}} = \sqrt{\frac{KD}{4}}$$

2. cos'è y_0

$$\langle V_x \rangle = \frac{u_*}{k} \ln \frac{y u_*}{\nu} + A$$

to be specified

3. cosa significa?

$$\exists\ y = \alpha D \ \text{t.c.}\ \langle V_x\rangle(y) = V$$

$\alpha + \ln A$

$$\frac{V}{u_*} = \frac{1}{k}\ln\frac{\alpha y u_*}{\nu} + A = \frac{1}{k}\ln \beta \frac{D u_*}{\nu}$$

exp. ≈1.13

$$C \triangleq \frac{V}{u_*} \qquad C = \frac{1}{k}\ln\left(\beta \frac{DV}{\nu}\cdot\frac{1}{C}\right)$$

calcolo esplicito di

$$C = \frac{1}{k}\ln\left(\beta\frac{Re}{C}\right)$$

moto TBL P.L.

$$V = C\sqrt{\frac{R}{2}\left(-\frac{dp}{dx}\frac{1}{\rho}\right)} = C\sqrt{\frac{D}{4}K}$$

u_* K

Poiseuille $V = \dfrac{KD^2}{32\nu} \rightarrow K = \dfrac{32\nu V}{D^2}$

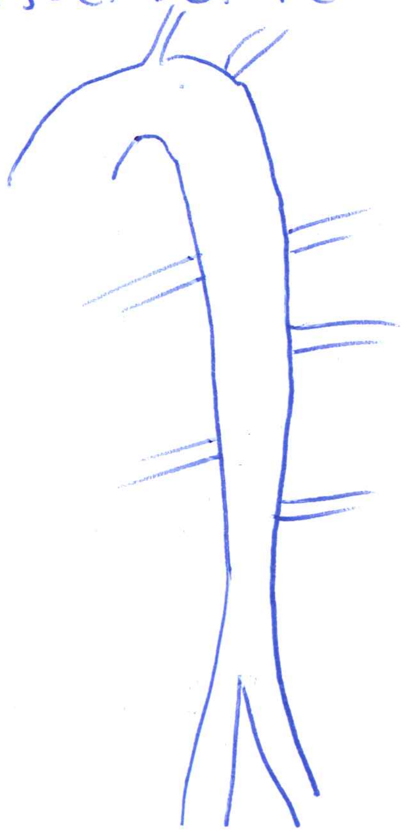
generale $V^2 = C^2\dfrac{D}{4}K \rightarrow K = \dfrac{4V^2}{C^2 D}$

$$C = \sqrt{\frac{Re}{8}}$$

moto Poiseuille

TAPERING

AORTA DISCENDENTE



Approccio condotto rigido

$$\boxed{\frac{dQ}{dx}=0}$$ SENZA USCITE LAT

$$A\frac{\partial V}{\partial x}+V\frac{\partial A}{\partial x}=0$$

$$\frac{\partial V}{\partial x}=-\frac{Q}{A^2}\frac{\partial A}{\partial x}>0$$

AUMENTEREBBE TROPPI ATTRITI

IN REALTA $\frac{dQ}{dx}<0$ PER DIRAMAZIONI

E ANCHE $\frac{dV}{dx}<0$ PER RIDURRE ATTRITI

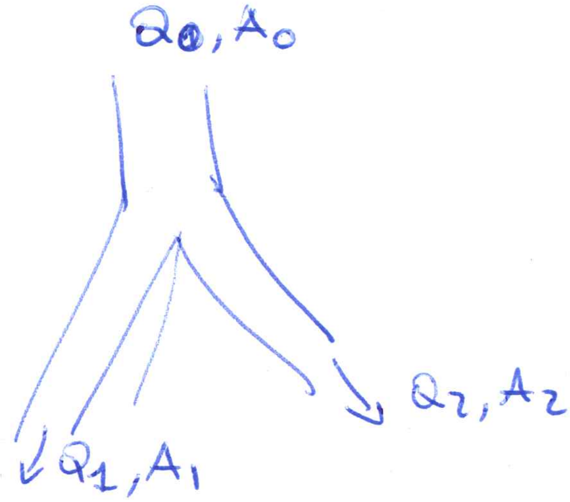
$$\frac{dQ}{dx}=A\frac{dV}{dx}+V\frac{dA}{dx} \Rightarrow \frac{dV}{dx}=\frac{1}{A}\frac{dQ}{dx}-\frac{V}{A}\frac{dA}{dx}<0$$

$$-\frac{1}{Q}\frac{dQ}{dx} > -\frac{1}{A}\frac{dA}{dx}$$

RIDUZ.% DI PORTATA > RIDUZ PERC SEZIONE

QUINDI NE È USCTO DI PIÙ

Biforcaz.



$$m_2 \quad V_1, V_2 < V_0$$

$$V_0 A_0 = V_1 A_1 + V_2 A_2$$ cons. massa

$$V_1 = V_2 \ , \ A_1 = A_2 \quad (sym.)$$

$$V_0 A_0 = \cancel{2} V_1 (A_1 \cdot 2)$$

$$\frac{V_0}{V_1} = \frac{\cancel{2} A_1 + A_2}{A_0} > 1 \qquad A_{valle} > A_{monte}$$

$$A_{AORTA} \simeq \pi \frac{3^2}{4} \simeq 7 cm^2$$

$$A_{TOT\ CAPILLAR} \simeq 1 m^2 !$$

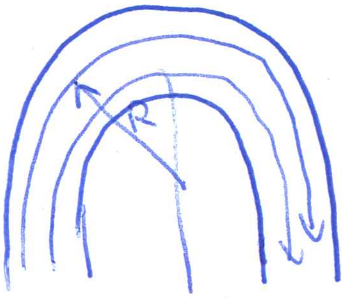
$$V_{AO} \simeq 1 m/s \qquad Q = 100 \times 7 \simeq 700 \ cm^3/s$$

$$V_{CAPILL} < 1 mm/s \qquad = 0,1 \times 7000$$

$$= 0,07 \times 100,00$$

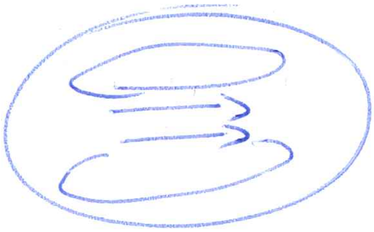
S40

VASI CON CURVATURA



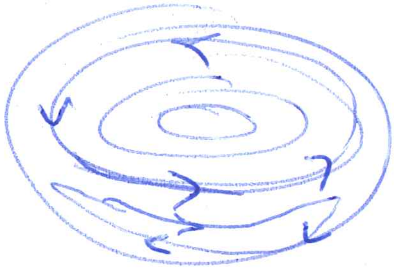
il moto non può essere "parallelo
che traj interne + corte
di quelle esterne

higher centrifugal acceleration $\frac{v^2}{R}$



CIRCOLAZ. SECONDARIE

Se non perfett. planare (elicoidale) cells are unequal



helical flow

S41

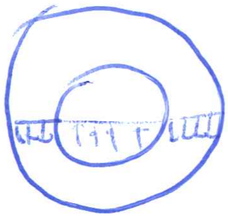
helical flow important no stagnation!

S42-S44

$\boxed{S45}$

CONDOTTI ELASTICI

Mecc. globale: def. Arteria soggetta a Δp



EQ. "MOTO" EQUILIBRIO

$$\Delta p \cdot \cancel{2} R = \sigma \cdot \cancel{2} s$$

$\boxed{S46}$

EQ. COSTITUTIVA

$$\sigma = E \frac{\Delta R}{R}$$

$$\Rightarrow \quad \frac{\Delta p R}{E s} = \frac{\Delta R}{R} \quad \Longleftrightarrow \quad \frac{\Delta A}{A} = \frac{\cancel{\pi} 2 \cancel{R} \Delta R}{\cancel{\pi} R^2} = \frac{\Delta p D}{E s}$$

Cons. massa (s piccolo)

$$\cancel{2\pi} R \cdot s = \cancel{2\pi} (R + \Delta R)(R + \Delta s)$$

$$\Rightarrow \frac{\Delta s}{s} \approx - \frac{\Delta R}{R}$$

more generally....

Flusso in condotti elastici — wave prop. — 547 - 548

Eq. cont. $\frac{\partial A}{\partial t} + v \frac{\partial A}{\partial x} + A \frac{\partial v}{\partial x} = 0$

Eq. moto $\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \text{loss} \simeq 0$

Eq. elastic. $A(p)$ such that $\frac{\partial A}{\partial \cancel{x}} = \frac{\partial A}{\partial p} \frac{\partial p}{\partial \cancel{x}}$

$$\frac{dA}{A} = \frac{dp\, D}{Es} \Rightarrow \frac{dA}{dp} = \frac{AD}{Es}$$

HP fenomeno prop. con celerità $c \gg v$

$$\Rightarrow \frac{\partial}{\partial t} \gg v \frac{\partial}{\partial x}$$

verificheremo a posteriori

$$\begin{cases} \frac{\partial A}{\partial t} + A \frac{\partial v}{\partial x} = 0 & \text{CONT} \\ \frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 & \text{MOTO} \end{cases} \xRightarrow{A(p)} \begin{cases} \frac{1}{A} \frac{\partial A}{\partial p} \frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} = 0 \\ \frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \end{cases}$$

$$\frac{\partial}{\partial t} \text{CONT} - \frac{\partial}{\partial x} \text{MOTO}$$

HP $\frac{1}{A} \frac{\partial A}{\partial p}$ slowly varying

$$\frac{1}{A} \frac{\partial A}{\partial p} \frac{\partial^2 p}{\partial t^2} - \frac{1}{\rho} \frac{\partial^2 p}{\partial x^2} = 0$$

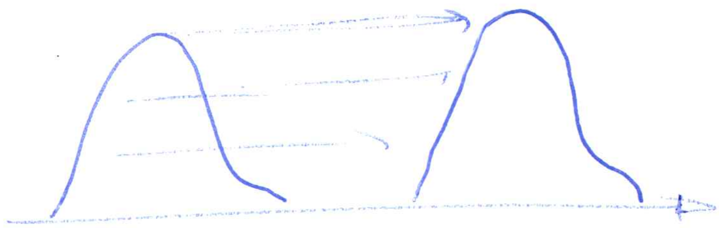
$$\Downarrow$$

$$\boxed{\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial^2 p}{\partial x^2} = 0}$$

WAVE EQ.

$$c = \sqrt{\frac{A}{\rho} \frac{dp}{dA}} \simeq \sqrt{\frac{Es}{\rho D}}$$

solution $\quad p(x,t) = p(x \pm ct)$



Propagation with same shape

$$c = \sqrt{\frac{A}{\rho}\frac{dp}{dA}} \underset{\text{DEF}}{\overset{\text{SMALL}}{=}} \sqrt{\frac{Es}{\rho D}}$$

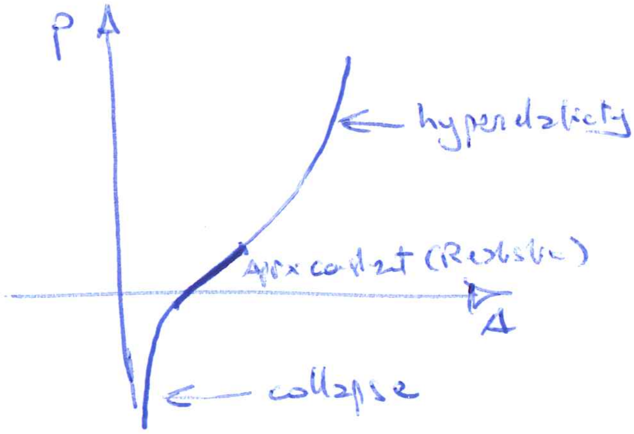
AORTA $\quad E \simeq 10^5 \frac{N}{m^2} \quad \frac{s}{D} \simeq 0,1 \Rightarrow c \simeq 3 \div 5\ m/s$

increases $\sim 10\ m/s$ peripheral arteries

A way to measure **E**

Approfondimento

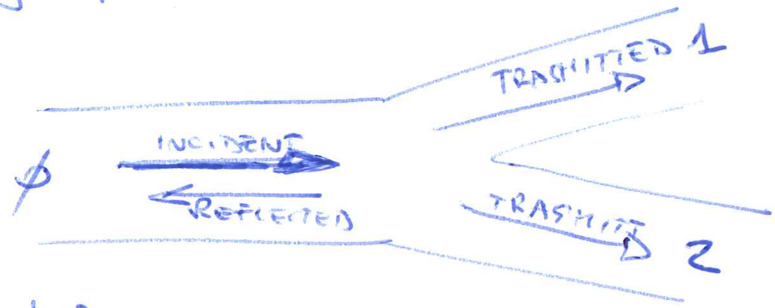
TUBE LAW



- Friction $\neq 0$ $\Rightarrow$ Attenuation the wave smooth-out
- c higher at higher p $\Rightarrow$ Irripidimento fronte (poi attenuato)
- Vessel not uniform (Aorta is tapered and stiffer) irripidimento picco
- junctions?

TRASMISSION / REFLECTION

At every bifurcation wave is Transmitted downstream + partially Reflected up



To quantify:

At the junction (cont. of pressure) ~ eq. moto $\quad P_I + P_R = P_{T_1} = P_{T_2}$

Continuity $\quad Q_I - Q_R = Q_{T_1} + Q_{T_2}$

let's find a relationship between Q & p

consider
$$p = A_p \sin(\omega(x-ct)+\varphi)$$
$$u = A_u \sin(\omega(x-ct)+\varphi')$$

take either CONT or MOTO

$$\cancel{\frac{1}{\rho c}\frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} = 0} \qquad \frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \quad \Rightarrow A_u = \frac{A_p}{\rho c}$$

$$\Rightarrow Q = Au = \frac{A}{\rho c} p = \frac{p}{Z} \leftarrow \text{IMPEDENZA } \frac{\rho c}{A}$$

Continuity
$$\begin{cases} \dfrac{P_I - P_R}{Z_0} = \dfrac{P_{T_1}}{Z_1} + \dfrac{P_{T_2}}{Z_2} \\ P_I + P_R = P_{T_1} \\ P_I + P_R = P_{T_2} \end{cases}$$

$$\frac{P_I - P_R}{Z_0} = \frac{P_I + P_R}{Z_1} + \frac{P_I + P_R}{Z_2}$$

$$P_I\left(\frac{1}{Z_0} - \frac{1}{Z_1} - \frac{1}{Z_2}\right) = P_R\left(\frac{1}{Z_0} + \frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

coeff. riflett. $$R = \frac{P_R}{P_I} = \frac{\frac{1}{Z_0} - \frac{1}{Z_1} - \frac{1}{Z_2}}{\frac{1}{Z_0} + \frac{1}{Z_1} + \frac{1}{Z_2}} \xrightarrow[Z_1 = Z_2]{sym} \frac{Z_1 - 2Z_0}{Z_1 + 2Z_0}$$

coeff. trasm. $$T = \frac{P_T}{P_I} = 1 + R$$

---

if. $\frac{1}{Z_1} + \frac{1}{Z_2} \simeq \frac{1}{Z_0}$ (i.e. sym. $Z_1 = 2Z_0$)

$$\Rightarrow R \simeq 0$$

$$\Rightarrow \frac{A_1}{\rho c_1} + \frac{A_2}{\rho c_2} = \frac{A_0}{\rho c_0} \quad ; \quad A_1\sqrt{\frac{D_1}{\rho E_1 S_1}} + A_2\sqrt{\frac{D_2}{\rho E_2 S_2}} = A_0\sqrt{\frac{D_0}{\rho E_0 S_0}}$$

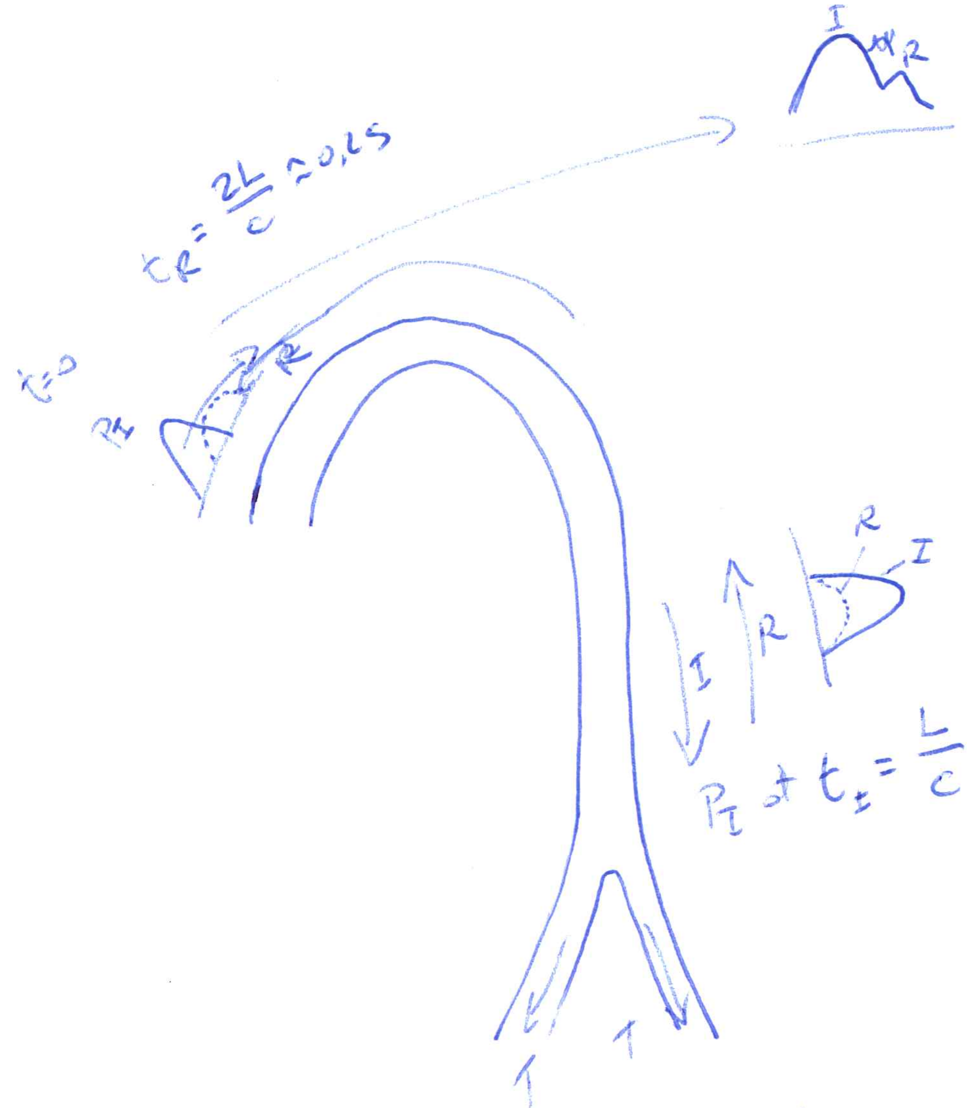
if. $E_1 = E_2 = E_0$ & $\frac{S_1}{D_1} = \frac{S_2}{D_2} = \frac{S_0}{D_0} \Rightarrow A_0 = A_1 + A_2$

$\Rightarrow V_0 = V_1 = V_2$

usually $E_1, E_2 > E_0$ & $\frac{S_1}{D_1}, \frac{S_2}{D_2} > \frac{S_0}{D_0}$

$\Rightarrow c_1, c_2 > c_0 \Rightarrow A_1 + A_2 > A_0$

$R \neq 0$ although small



$t_R = \frac{2L}{c} \simeq 0{,}2\,s$

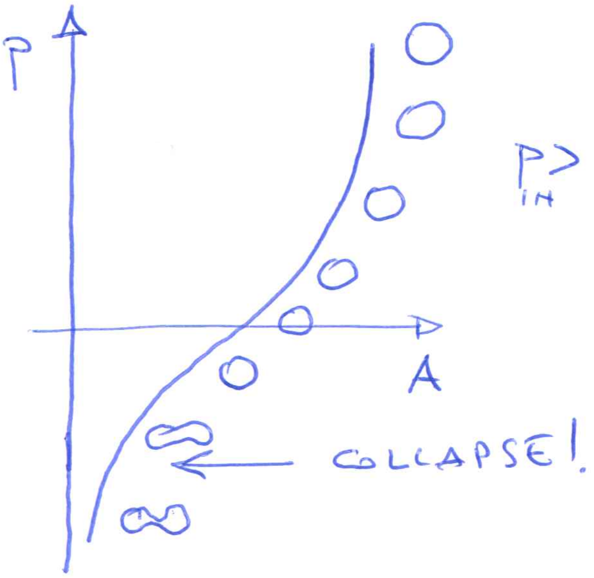
P_I at $t_1 = \frac{L}{c} \simeq \frac{0.5\,m}{5\,m/s} \simeq 0{,}1\,s$

Bce pressure is thought to help coronary flow

~~A proposito delle impedenze~~ . . .

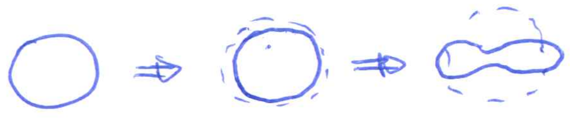
More important is the backflow [illegible] α is large!

COLLAPSIBLE VESSEL



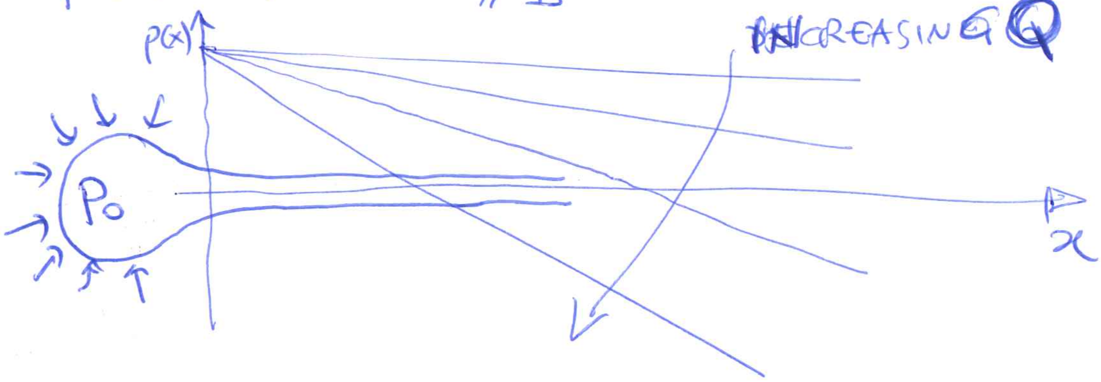
$P_{IN} > P_{EXT}$

$P_{INTERNA} < P_{EXTERNA}$



~~Poiseuille~~ Poiseuille $Q = \frac{\pi}{128\mu} \frac{dp}{dx} D^4$

$$\Rightarrow p(x) = p_0 - \frac{128 \mu Q}{\pi D^4} x$$



$p(x) < P_{EXT} \Rightarrow$ collapse

$\Rightarrow$ Q cannot increase

"FLOW LIMITATION"

- Giraffe jug. vein
- (soffio) forte espirazione
- uretere (minzione soprattutto)

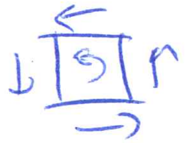
S50-S53

9 VORTICITÀ

1

- limiti definizione velocità
  OK x flussi idrodinamici ma non x moti complessi S54

- Def. vorticità $\omega = +\nabla \times \omega$



elementi: vortice = zeroth... of ω

$$\Gamma = \int \omega\, dA = \oint v \cdot ds$$

shear layer = b. l.! S55

$$\Delta v = \gamma = \int \frac{\partial \omega}{\partial n}\, dn = \int \omega\, ds$$

- Decomposizione stokes

$$\bar{u} = \nabla \varphi + \nabla \times \bar{\psi} \quad \begin{cases} \nabla \cdot \bar{u} = \nabla^2 \varphi & \text{MASS} \\ \nabla \times \bar{u} = \nabla^2 \psi & \text{VORTICITY} \end{cases}$$

N.B. $\nabla^2 u = \underbrace{\nabla^2 (\nabla \varphi)}_{\text{ZERO}} + \nabla^2 (\bar{\nabla} \times \psi)$

IRROTATIONAL PART IS "~~IDEAL~~" NO "REAL"
JUST adj mass balance

- Eq. ~~VO~~ VORTICITÀ

$$\nabla \times NS \Rightarrow \frac{\partial \omega}{\partial t} + \nabla \times \underbrace{(v \cdot \nabla v)}_{v \cdot \nabla \omega - \omega \cdot \nabla v} = \nu \nabla^2 \omega \quad (*)$$

$$= \frac{\partial}{\partial y}\left(v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) - \frac{\partial}{\partial z}\left(v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) =$$

$$= v_x \frac{\partial}{\partial x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + v_y \frac{\partial}{\partial y}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + v_z \frac{\partial}{\partial z}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) +$$

$$\frac{\partial v_x}{\partial y}\frac{\partial v_z}{\partial x} + \underbrace{\frac{\partial v_y}{\partial y}}_{-\frac{\partial v_x}{\partial x} - \frac{\partial v_z}{\partial z}}\frac{\partial v_z}{\partial y} + \frac{\partial v_z}{\partial y}\frac{\partial v_z}{\partial z} - \frac{\partial v_x}{\partial z}\frac{\partial v_y}{\partial x} - \frac{\partial v_y}{\partial z}\frac{\partial v_y}{\partial y} - \frac{\partial v_z}{\partial z}\frac{\partial v_y}{\partial z} =$$

$$= v_x \frac{\partial \omega_x}{\partial x} + v_y \frac{\partial \omega_x}{\partial y} + v_z \frac{\partial \omega_x}{\partial z} +$$

$$\frac{\partial v_x}{\partial y}\frac{\partial v_z}{\partial x} + \frac{\partial v_z}{\partial y}\overbrace{\left(-\underset{\mathrm{I}}{\frac{\partial v_x}{\partial x}} - \cancel{\frac{\partial v_z}{\partial z}}\right)}^{\frac{\partial v_y}{\partial y}} + \cancel{\frac{\partial v_z}{\partial y}\frac{\partial v_z}{\partial z}}$$

$$- \frac{\partial v_x}{\partial z}\frac{\partial v_y}{\partial x} - \frac{\partial v_y}{\partial z}\overbrace{\left(-\underset{\mathrm{I}}{\frac{\partial v_x}{\partial x}} - \cancel{\frac{\partial v_z}{\partial z}}\right)}^{\frac{\partial v_y}{\partial y}} - \cancel{\frac{\partial v_z}{\partial z}\frac{\partial v_y}{\partial z}} =$$

$$= v \cdot \nabla \omega_x - \frac{\partial v_x}{\partial x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) - \frac{\partial v_x}{\partial y}\left(-\frac{\partial v_z}{\partial x}\right)$$

$$- \frac{\partial v_x}{\partial z}\left(\frac{\partial v_y}{\partial x}\right)$$

ADD ZERO

$$+ \frac{\partial v_x}{\partial y}\frac{\partial v_x}{\partial z} - \frac{\partial v_x}{\partial y}\frac{\partial v_x}{\partial z}$$

$$= v \cdot \nabla \omega_x - \omega \cdot \nabla v_x$$

$$= v \cdot \nabla \omega \Big]_x - \omega \cdot \nabla v \Big]_x$$

EQ. VORT. (N.S. – Term of ω "non trivial")

$$\boxed{\frac{\partial \omega}{\partial t} + v \cdot \nabla \omega = \omega \cdot \nabla v + \nu \nabla^2 \omega}$$

$\omega(0) = 0 \Rightarrow \omega(t) = 0$ GENERATION ! Helmholtz law 1 (INVISCID)

$\boxed{\text{DINAMICS}}$

TRASPORT $\frac{d\omega}{dt} = 0$ H. law 2

STRETCHING. 3D only !

H. law 3

$\nabla \cdot v = 0$

Γ conserved

DIGRESSION 2D PATCHES V. LAYERS... ONLY

DIFFUSION ‡ high ∇ SMALL SCALES .. CREATED by STRETCH.

VISC. RECONNECT.

B.L. SEPARATION

4

ω-generation ⇒ ADERENZA THEN DIFFUSION

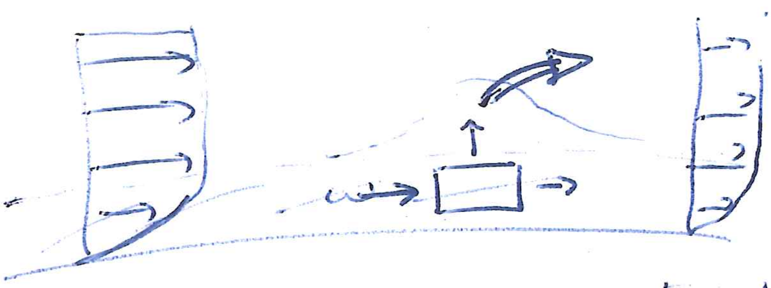
b.l. width ≡ DIFFUSION $h \approx \sqrt{\nu t}$

example

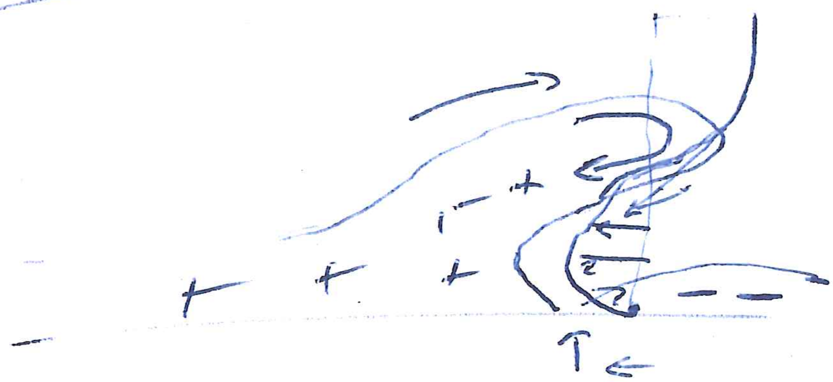
$\vec{\tau}_w = \mu \vec{\omega}$

SEPARATION ⟺ DECELERATION

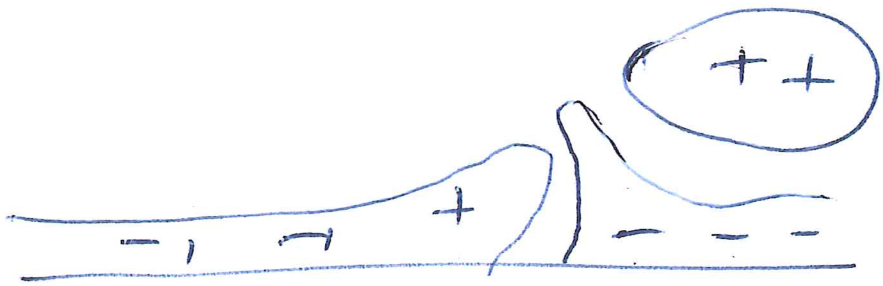
S56



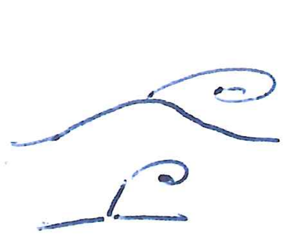
NEG. ∇p



DECEL.



examples



⇒

VORTEX INDUCED

Vortex formation

Sep. ω-layer ⇒ roll-up   S57 - S61

in the. instability of shear-layer (b.l.)

sharp-edge ; smooth surface $\boxed{S62\text{-}S64}$

— EXAMPLES —   K-H instability

Sep. is 3D ! 3 vortex if 2 tube

RING (AXISYMM.)

$\underset{V/2}{\rightarrow}$ ≠

$$\frac{d\Gamma}{dt} = \underset{\omega\, ds}{\gamma} \cdot \overset{(\Delta V)}{\underset{ds/dt}{\frac{V}{2}}} \approx \frac{V^2}{2}$$

self induced vel. $V_\Gamma \simeq \frac{\Gamma}{4\pi R}\left(\ln\frac{R}{e} \pm \frac{1}{2}\right)$

when $V_\Gamma > \frac{V}{2}$ ⇒ DETACHES

$$\Rightarrow \frac{\Gamma}{4\pi R}(\,) > \frac{V}{2} \qquad \frac{V^2 t}{2}\,\frac{(\,)}{4\pi R} > \frac{V}{2}$$

$$(\mathrm{VFT}) \Rightarrow \frac{Vt}{D} > \frac{2\pi}{(\,)} \simeq 4$$

$\boxed{S65\text{-}S66}$ $\boxed{S68}$

VORTEX FORCE

ADDITIONAL $\Delta p \sim \frac{\partial \Gamma}{\partial t}$ TRANSF. OF ENERGY INTO VORTEX INERTIA

V-V & V-wall interactions

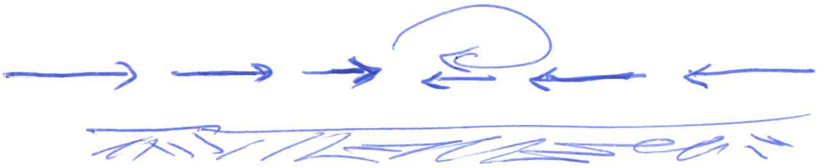
2D ⊕⊕ MERGE   with wall   vortex-induced separation $\boxed{S69\text{-}S71}$

⊕⊖ roll-up ANNIHILATION

3D ——— stretching --- breakup   TBL

10 moto con separazione nei grandi vasi

WSS & development of arteriosclerosis



low WSS (time mean) $\Rightarrow$ lack of alignment of endo-cells, deposition of material

$$AWSS = \frac{1}{T}\int_0^T \tau \, dt$$

oscillating WSS $\Rightarrow$ disturbing endo-cells

$$OSI = \frac{1}{2}\left(1 - \frac{\left|\int_0^T \tau \, dt\right|}{\int_0^T |\tau| \, dt}\right) \begin{cases} \to 0 & \text{ok} \\ \to \frac{1}{2} & \text{dang.} \end{cases}$$

QUALITATIVELY

high WSS gradients / anomalous WSS

N.B. Vortex formation $\simeq$ anomalous WSS

$\Rightarrow$ artherosclerosis risk

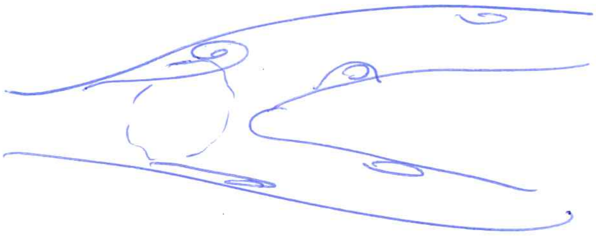
Moto con separazione nei grandi vasi

Casi di separazione:

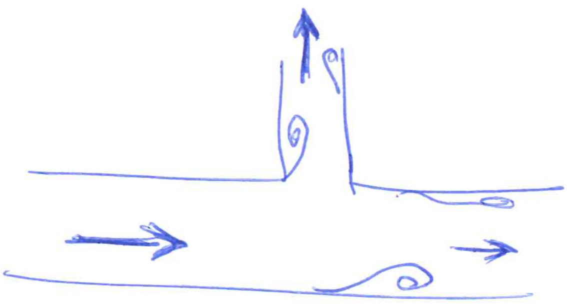
Narrowing (stenosis)

enlargement (aneurysm)

biforcazioni (carotid iliac)



ramificazioni



confluenze

Valves

Stenosis

S73

definitions:
narrowing

atherogenesis

small disturbances
→ + deposition
→ ++ deposition
→ blockage

S74

self-sustained
rare (or none) reduction!

Typical sites:

carotid (blood to brain) S75

consequences → ictus! death!

therapy: fluidificanti (per non farla procedere)

surgery:
1. atherectomy S76
2. stenting (endovascular prosthesis) S77 S78

coronaries (blood to heart muscle) $\boxed{S79-S80}$

consequences → ischemia, infarction, death! $\boxed{S81}$

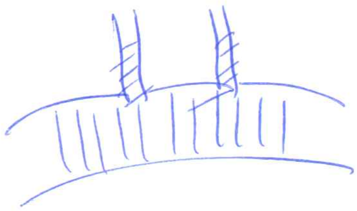
therapy: fluidificanti (or no facile procedure)

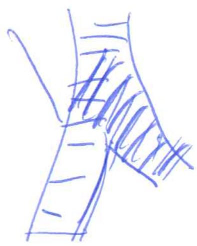
surgery:
- ① by-pass $\boxed{S82}$
- ② stenting $\boxed{S83}$

other places

aorta, iliac arthery
aortic ramifications

large variety of stents





S84-85

Aneurismi

local enlargement of a vessel

side enlargement

inline enlargement

Fluid dyn. phenomena → stagnation areas
→ splash: hammering

~~development~~ birth due to local tissue weakening/defection

development due to continuing cause

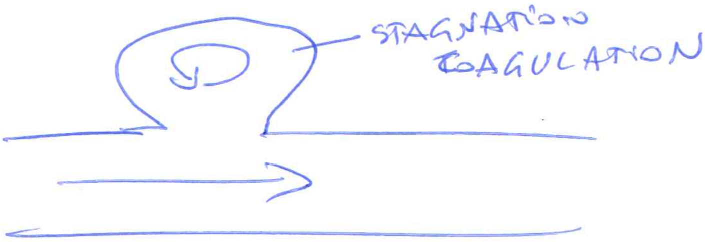
~~risk~~ risk = rupture
- itching
- hemorrhage
- death (SUDDEN!)

typical sites

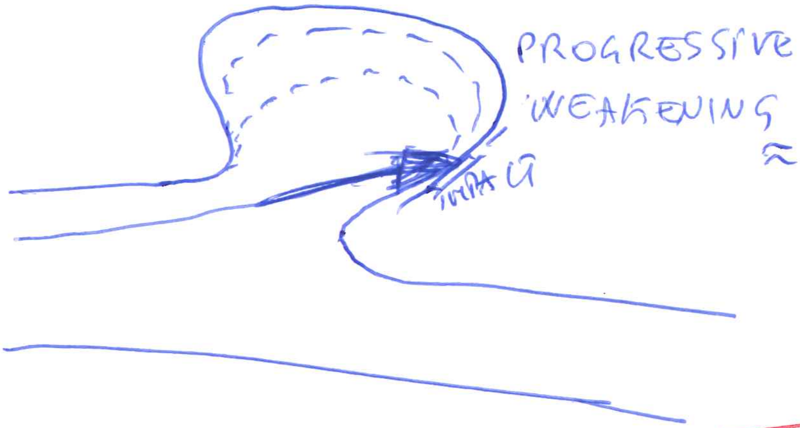
AORTA
- ROOT ← VALVE SET, GENETIC
- ARCH
- ABDOMINAL (iliac bifurcation) AAA

brain

Role of fluid dynamics

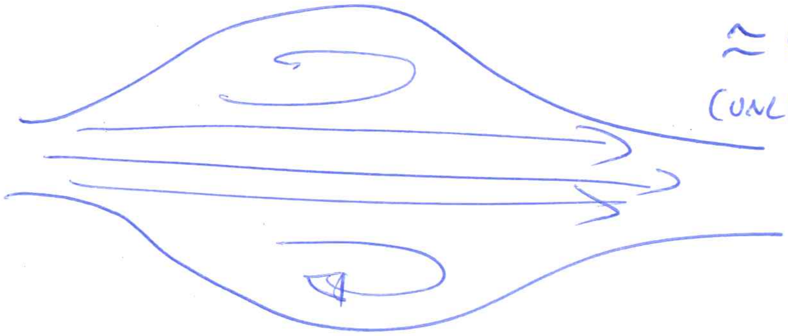


≈ STABLE?



≈ UNSTABLE?

S87-89



≈ STABLE?
(UNLIKELY SYMMETRIC)

≈ UNSTABLE

THERAPY ≈ NONE   frequent controls to monitor — NOT EASY
ECHO/MR!

SURGERY — OPEN — BENDAGE
— REPLACEMEN
— STENTING

Capitoli 11 e 12

(Meccanica Cardiaca)

in preparazione